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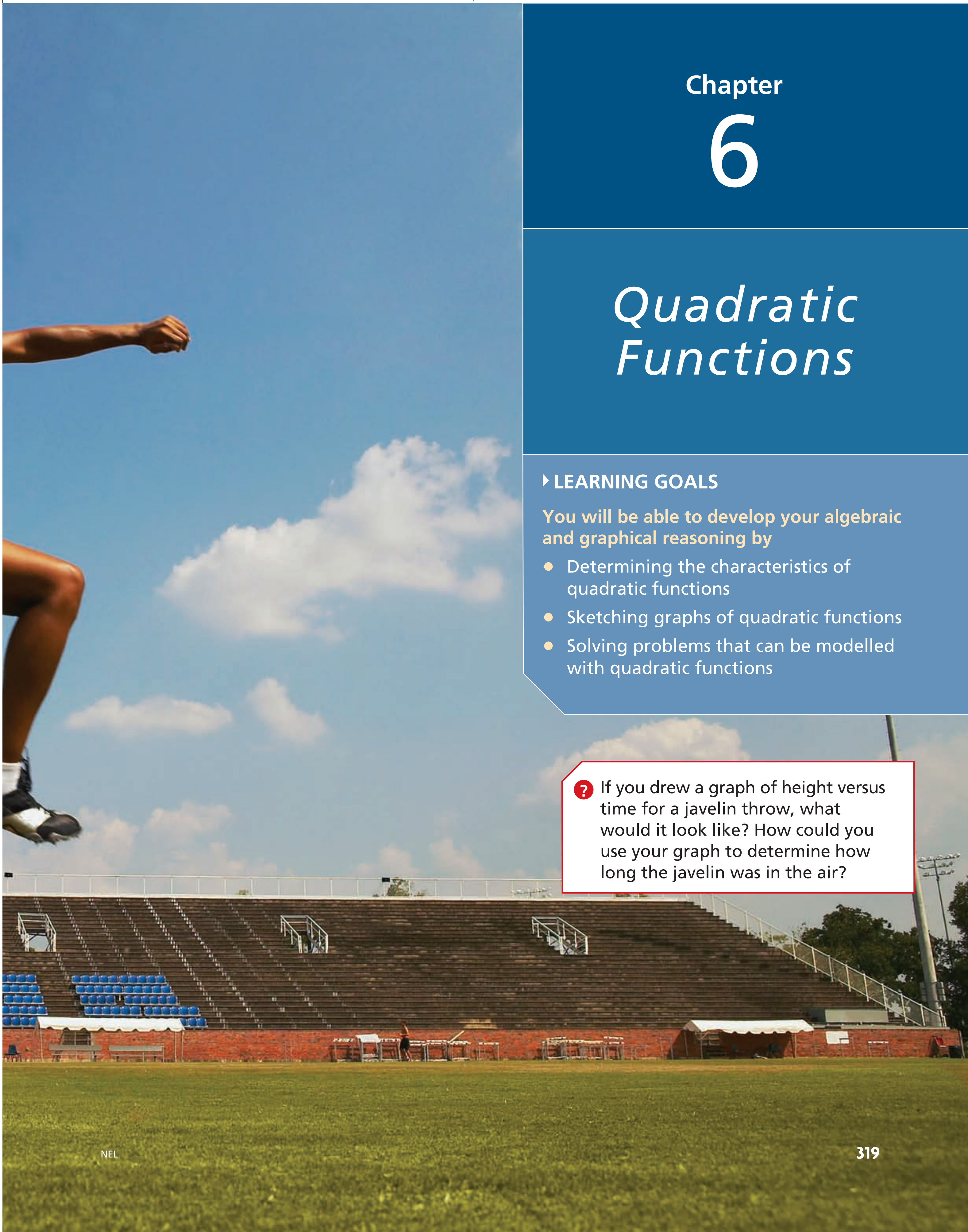

Chapter

6

Quadratic Functions

LEARNING GOALS

You will be able to develop your algebraic and graphical reasoning by

- Determining the characteristics of quadratic functions
- Sketching graphs of quadratic functions
- Solving problems that can be modelled with quadratic functions

? If you drew a graph of height versus time for a javelin throw, what would it look like? How could you use your graph to determine how long the javelin was in the air?

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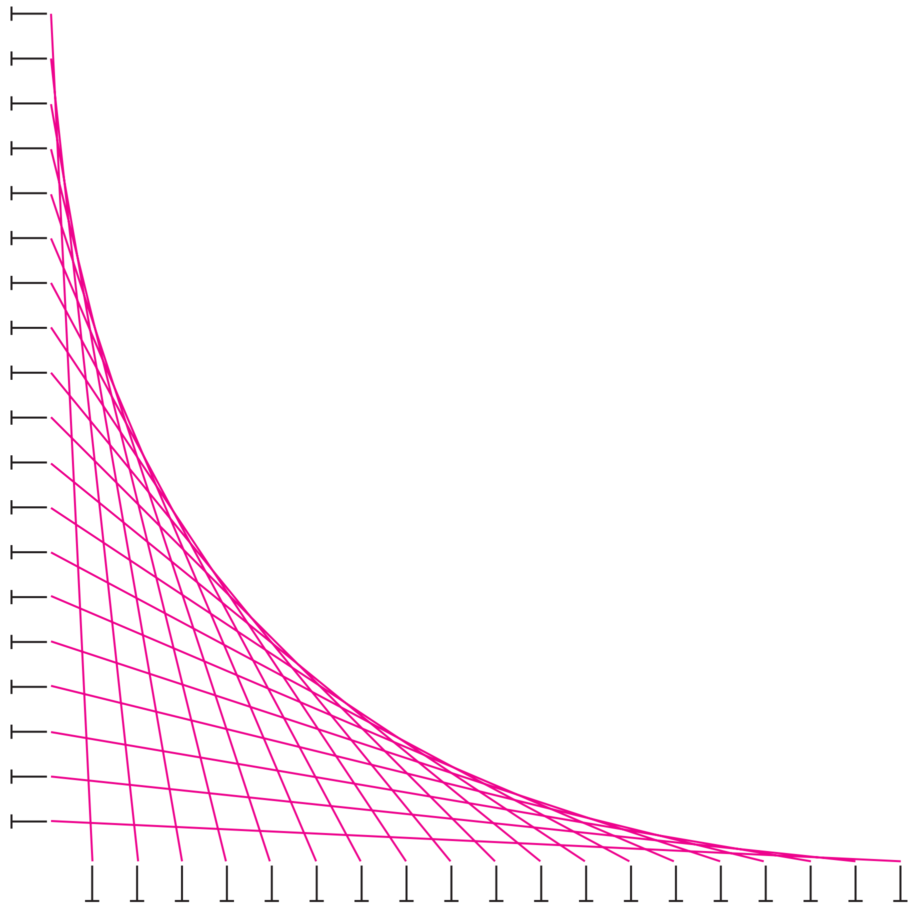
6 Getting Started

String Art

Robert's grandmother is teaching him how to make string art. In string art, nails are spaced evenly on a board and connected to each other by lines of string or yarn. The board that Robert is using has an array of 38 nails: 19 placed horizontally and 19 placed vertically as shown. The nails are 1 cm apart. Robert started to make the artwork and noticed that the lines of yarn were different lengths.



? Use a model to describe the relation between the position of each nail and the length of the yarn that connects it to another nail.

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A. Model Robert's art on a coordinate grid, using line segments to represent the pieces of string.

B. Determine the length of each line segment.

C. Create a table of values like the one shown to compare nail position with string length.

Nail Position, x	String Length, y
1	
2	
3	

D. Describe any patterns you see in your table of values.

E. Is the relation linear? Explain.

F. Determine the domain and range for the relation.

G. Graph the relation.

H. What conclusions about string length can you make from your models?

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. Graphs of functions are straight lines.
2. Functions are not symmetrical.
3. If the domain of a function is the set of real numbers, then its range will also be the set of real numbers.

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6.1 Exploring Quadratic Relations

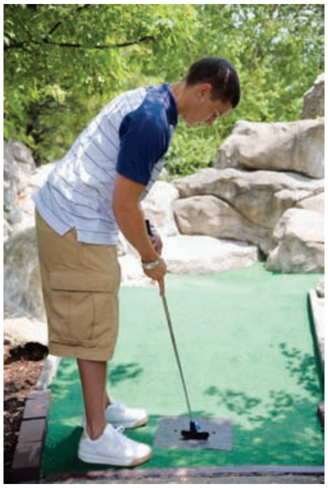
YOU WILL NEED

- graphing technology

GOAL

Determine the characteristics of quadratic relations.

EXPLORE the Math



A moving object that is influenced by the force of gravity can often be modelled by a **quadratic relation** (assuming that there is no friction). For example, on one hole of a mini-golf course, the ball rolls up an incline after it is hit, slowing all the way due to gravity. If the ball misses the hole, it rolls back down the incline, accelerating all the way. If the initial speed of the ball is 6 m/s, the distance of the ball from its starting point in metres, y, can be modelled by the quadratic relation

$$y = -2.5x^2 + 6x$$

where x is the time in seconds after the ball leaves the putter.

quadratic relation
A relation that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$; for example, $y = 4x^2 + 2x + 1$

? How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

Reflecting

A. Describe the common characteristics of each of the **parabolas** you graphed.

parabola
The shape of the graph of any quadratic relation.

B. Describe any symmetry in your graphs.

C. Are the quadratic relations that you graphed functions? Justify your decision.

D. What effects do the following changes have on a graph of a quadratic relation?

i) The value of a is changed, but b and c are left constant.
ii) The value of b is changed, but a and c are left constant.
iii) The value of c is changed, but a and b are left constant.

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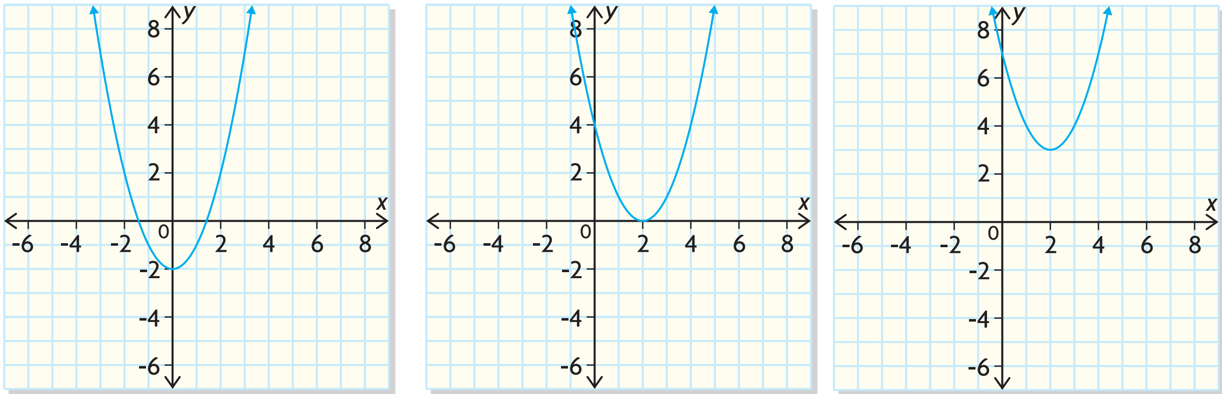
E. The graphs of three quadratic relations are shown. Predict possible values of a, b, and c in the equation for each graph.



In Summary

Key Ideas

- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

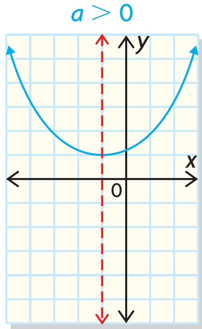
$$y = ax^2 + bx + c$$

  where $a \neq 0$.
- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

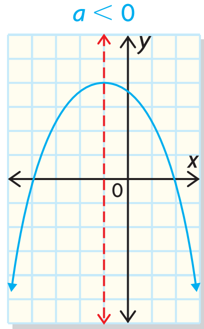
Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry.
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.

$a > 0$



$a < 0$



  - Changing the value of b changes the location of the parabola's line of symmetry.
  - The constant term, c, is the value of the parabola's y-intercept.

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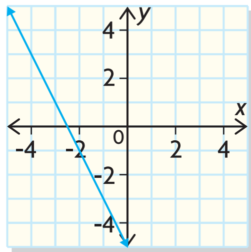
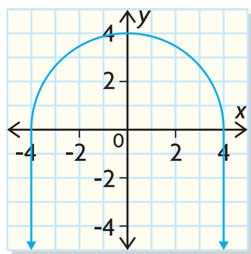
FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.

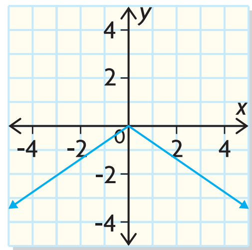
a)



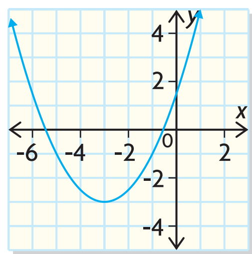
b)



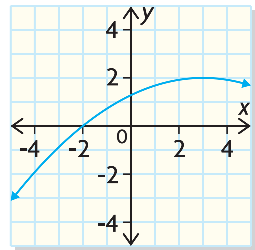
c)



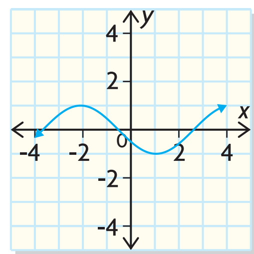
d)



e)



f)



2. Which of the following relations are quadratic? Explain.

a) $y = 2x - 7$
b) $y = 2x(x + 3)$
c) $y = (x + 4)^2 + 1$
d) $y = x^2 - 5x - 6$
e) $y = 4x^3 + x^2 - x$
f) $y = x(x + 1)^2 - 7$

3. State the y-intercept for each quadratic relation in question 2.

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$
b) $y = -2x^2 + 6x$
c) $y = 9 - x + 3x^2$
d) $y = -\frac{2}{3}x^2 - 6x + 1$

6. Each table of values lists points in a quadratic relation. Decide, without graphing, the direction in which the parabola opens.

a)

x	−4	−3	−2	−1	0	1
y	12	5	0	−3	−4	−3

b)

x	0	1	2	3	4	5
y	−13	−3	3	5	3	−3

c)

x	−5	−4	−3	−2	−1	0
y	3.0	−0.5	−3.0	−4.5	−5.0	−4.5

d)

x	0	1	2	3	4	5
y	−4	19	40	59	76	91

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6.2 Properties of Graphs of Quadratic Functions

GOAL

Identify the characteristics of graphs of quadratic functions, and use the graphs to solve problems.

YOU WILL NEED

- ruler
- graph paper

LEARN ABOUT the Math

Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor. Marko wanted to capture a photo of the ball at its greatest height. However, after looking at the photographs, he could not be sure that he had done so. He decided to place the information from his photographs in a table of values.

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

From his photographs, Marko observed that Nicolina struck the ball at a height of 2 ft above the ground. He also observed that it took about 1.25 s for the ball to reach the same height on the way down.

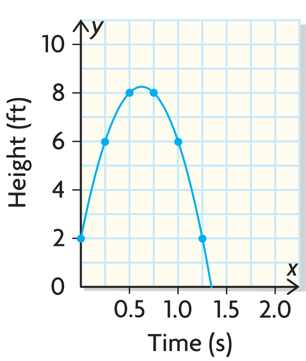
? When did the volleyball reach its greatest height?

EXPLORE...

- Parabolic skis are marketed as performing better than traditional straight-edge skis. Parabolic skis are narrower in the middle than on the ends. Design one side of a parabolic ski on a coordinate grid. In groups, discuss where any lines of symmetry occur and how the parabolic shape works in your design.

EXAMPLE 1 Using symmetry to estimate the coordinates of the vertex

Marko's Solution



I plotted the points from my table, and then I sketched a graph that passed through all the points.

The graph looked like a parabola, so I concluded that the relation is probably quadratic.

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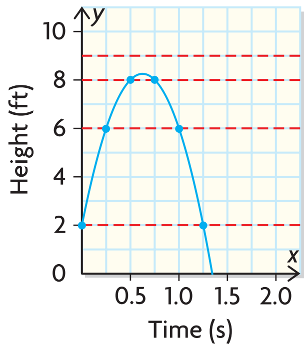
vertex
The point at which the quadratic function reaches its maximum or minimum value.



I knew that I could draw horizontal lines that would intersect the parabola at two points, except at the **vertex**, where a horizontal line would intersect the parabola at only one point.

Using a ruler, I drew horizontal lines and estimated that the coordinates of the vertex are around (0.6, 8.2).

This means that the ball reached maximum height at just over 8 ft, about 0.6 s after it was launched.

Equation of the axis of symmetry:

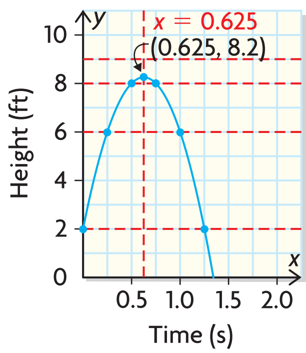
$$x = \frac{0 + 1.25}{2}$$

$$x = 0.625$$

I used points that have the same y-value, (0, 2) and (1.25, 2), to determine the equation of the **axis of symmetry**. I knew that the axis of symmetry must be the same distance from each of these points.

axis of symmetry
A line that separates a 2-D figure into two identical parts.

For example, a parabola has a vertical axis of symmetry passing through its vertex.



From the equation, the x-coordinate of the vertex is 0.625. From the graph, the y-coordinate of the vertex is close to 8.2.

Therefore, 0.625 s after the volleyball was struck, it reached its maximum height of approximately 8 ft 2 in.

I revised my estimate of the coordinates of the vertex.

Reflecting

A. How could Marko conclude that the graph was a quadratic function?

B. If a horizontal line intersects a parabola at two points, can one of the points be the vertex? Explain.

C. Explain how Marko was able to use symmetry to determine the time at which the volleyball reached its maximum height.

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APPLY the Math

EXAMPLE 2 Reasoning about the maximum value of a quadratic function

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of these jets forms an arch that can be defined by the function

$$f(x) = -0.12x^2 + 3x$$

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach?

Manuel's Solution

$$f(x) = -0.12x^2 + 3x$$

I knew that the degree of the function is 2, so the function is quadratic. The arch must be a parabola.

I also knew that the coefficient of x^2, a, is negative, so the parabola opens down. This means that the function has a **maximum value,** associated with the y-coordinate of the vertex.

$$f(0) = 0$$

I started to create a table of values by determining the y-intercept. I knew that the constant, zero, is the y-intercept. This confirms that the stream of water shoots from ground level.

I continued to increase x by intervals of 1 until I noticed a repeat in my values. A height of 18.72 ft occurs at horizontal distances of 12 ft and 13 ft.

$$f(1) = -0.12(1)^2 + 3(1)$$
$$f(1) = -0.12 + 3$$
$$f(1) = 2.88$$

x	0	1	2	12	13
$f(x)$	0	2.88	5.52	18.72	18.72

Based on symmetry and the table of values, the maximum value of $f(x)$ will occur halfway between (12, 18.72) and (13, 18.72).

The arch of water will reach a maximum height between 12 ft and 13 ft from the opening in the ground.

maximum value
The greatest value of the dependent variable in a relation.

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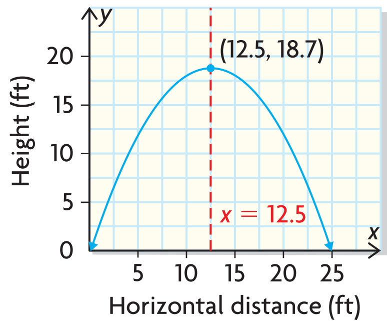

I used my table of values to sketch the graph. I extended the graph to the x-axis. I knew that my sketch represented only part of the function, since I am only looking at the water when it is above the ground.

$$x = \frac{12 + 13}{2}$$
$$x = 12.5$$

I used two points with the same y-value, (12, 18.72) and (13, 18.72), to determine the equation of the axis of symmetry.

Equation of the axis of symmetry:
$x = 12.5$

Height at the vertex:

$$f(x) = -0.12x^2 + 3x$$
$$f(12.5) = -0.12(12.5)^2 + 3(12.5)$$
$$f(12.5) = -0.12(156.25) + 37.5$$
$$f(12.5) = -18.75 + 37.5$$
$$f(12.5) = 18.75$$

I knew that the x-coordinate of the vertex is 12.5, so I substituted 12.5 into the equation to determine the height of the water at this horizontal distance.

The water reaches a maximum height of 18.75 ft when it is 12.5 ft from the opening in the ground.

The water can reach a maximum horizontal distance of 25 ft from the opening in the ground.

Due to symmetry, the opening in the ground must be the same horizontal distance from the axis of symmetry as the point on the ground where the water lands. I simply multiplied the horizontal distance to the axis of symmetry by 2.

The domain of this function is $0 \le x \le 25$, where $x \in R$.

Your Turn

Another water arch at the splash pad is defined by the following quadratic function:

$$f(x) = -0.15x^2 + 3x$$

a) Graph the function, and state its domain for this context.
b) State the range for this context.
c) Explain why the original function describes the path of the water being sprayed, whereas the function in *Example 1* does not describe the path of the volleyball.

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EXAMPLE 3 Graphing a quadratic function using a table of values

Sketch the graph of the function:

$$y = x^2 + x - 2$$

Determine the y-intercept, any x-intercepts, the equation of the line of symmetry, the coordinates of the vertex, and the domain and range of the function.

Anthony's Solution

$y = x^2 + x - 2$

The function is a quadratic function in the form

$ax^2 = bx + c$

$a = 1$

$b = 1$

$c = -2$

The degree of the given equation is 2, so the graph will be a parabola.

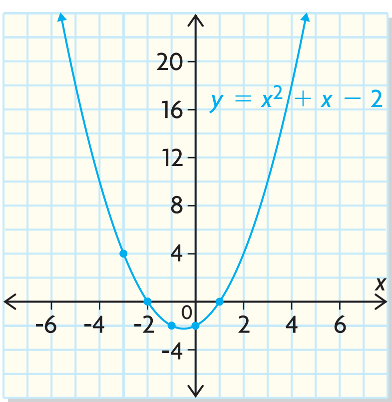
Since the coefficient of x^2 is positive, the parabola opens up.

Since the y-intercept is less than zero and the parabola opens up, there must be two x-intercepts and a **minimum value**.

x	−3	−2	−1	0	1
y	4	0	−2	−2	0

I made a table of values. I included the y-intercept, (0, −2), and determined some other points by substituting values of x into the equation.

I stopped determining points after I had identified both x-intercepts, because I knew that I had enough information to sketch an accurate graph.



I graphed each coordinate pair and then drew a parabola that passed through all the points.

Equation of the axis of symmetry:

$$x = \frac{-2 + 1}{2}$$

$$x = \frac{-1}{2}$$

$$x = -0.5$$

I used the x-intercepts to determine the equation of the axis of symmetry.

minimum value
The least value of the dependent variable in a relation.

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y-coordinate of the vertex:
$y = (-0.5)^2 + (-0.5) - 2$
$y = 0.25 - 0.5 - 2$
$y = -2.25$
The vertex is $(-0.5, -2.25)$.

I knew that the vertex is a point on the axis of symmetry. The x-coordinate of the vertex must be -0.5. To determine the y-coordinate of the vertex, I substituted -0.5 for x in the given equation.

The y-intercept is -2.
The x-intercepts are -2 and 1.
The equation of the axis of symmetry is $x = -0.5$.
The vertex is $(-0.5, -2.25)$.

The vertex, $(-0.5, -2.25)$, defines the minimum value of y.

Domain and range:
$\{(x, y) \mid x \in \mathrm{R}, y \geq -2.25, y \in \mathrm{R}\}$

No restrictions were given for x, so the domain is all real numbers.

Your Turn

Explain how you could decide if the graph of the function $y = -x^2 + x + 2$ has x-intercepts.

EXAMPLE 4 Locating a vertex using technology

A skier's jump was recorded in frame-by-frame analysis and placed in one picture, as shown.

The skier's coach used the picture to determine the quadratic function that relates the skier's height above the ground, y, measured in metres, to the time, x, in seconds that the skier was in the air:

$$y = -4.9x^2 + 15x + 1$$

Graph the function. Then determine the skier's maximum height, to the nearest tenth of a metre, and state the range of the function for this context.

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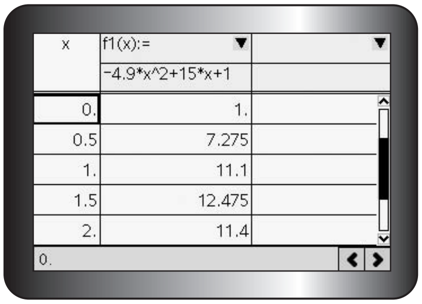
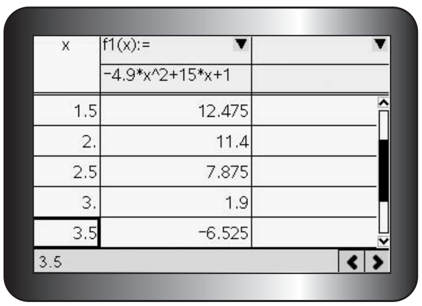
Isidro's Solution

$y = -4.9x^2 + 15x + 1$

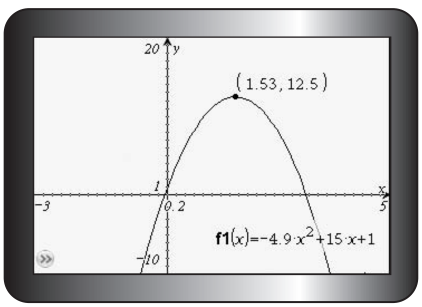




I entered the equation into my calculator.

To make sure that the graph models the situation, I set up a table of values. The skier's jump will start being timed at 0 s, and the skier will be in the air for only a few seconds, so I set the table to start at an *x*-value of zero and to increase in increments of 0.5.

I decided to set the minimum height at 0 m—it doesn't make sense to extend the function below the *x*-axis, because the skier cannot go below the ground. I checked the table and noticed that the greatest *y*-value is only 12.475... m, and that *y* is negative at 3.5 s. I used these values to set an appropriate viewing window for the graph.



I graphed the function and used the calculator to locate the maximum value of the function.

The skier achieved a maximum height of 12.5 m above the ground 1.5 s into the jump.

The range of the function is $\{y \mid 0 \leq y \leq 12.5, y \in \mathrm{R}\}$.

In this situation, the height of the skier varies between 0 m and 12.5 m.

Your Turn

On the next day of training, the coach asked the skier to increase his speed before taking the same jump. At the end of the day, the coach analyzed the results and determined the equation that models the skier's best jump:

$$y = -4.9x^2 + 20x + 1$$

How much higher did the skier go on this jump?

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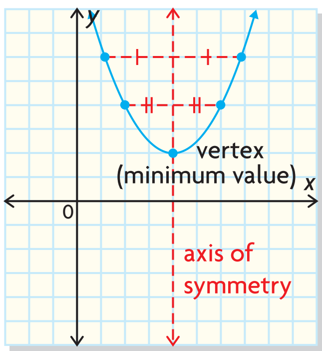
In Summary

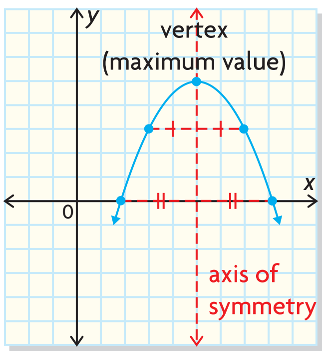
Key Idea

- A parabola that is defined by the equation $y = ax^2 + bx + c$ has the following characteristics:
  - If the parabola opens down ($a < 0$), the vertex of the parabola is the point with the greatest y-coordinate. The y-coordinate of the vertex is the maximum value of the function.
  - If the parabola opens up ($a > 0$), the vertex of the parabola is the point with the least y-coordinate. The y-coordinate of the vertex is the minimum value of the function.
  - The parabola is symmetrical about a vertical line, the axis of symmetry, through its vertex.

$y = ax^2 + bx + c$
$a > 0$

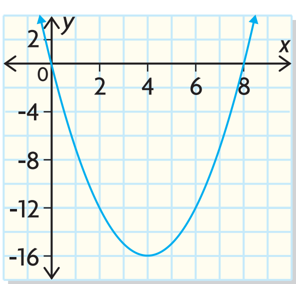


$y = ax^2 + bx + c,$
$a < 0$



Need to Know

- For all quadratic functions, the domain is the set of real numbers, and the range is a subset of real numbers.
- When a problem can be modelled by a quadratic function, the domain and range of the function may need to be restricted to values that have meaning in the context of the problem.



CHECK Your Understanding

1. **a)** Determine the equation of the axis of symmetry for the parabola.
b) Determine the coordinates of the vertex of the parabola.
c) State the domain and range of the function.

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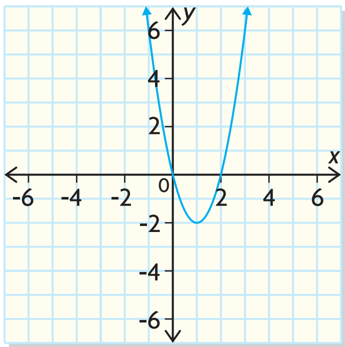
2. State the coordinates of the y-intercept and two additional ordered pairs for each function.

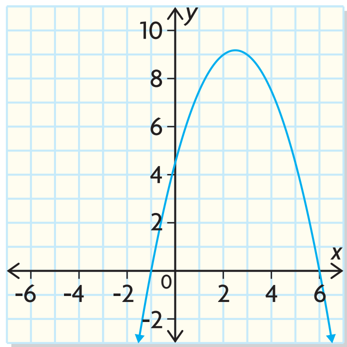
a) $f(x) = 2x^2 + 8x + 8$ **b)** $f(x) = 4x - x^2$

3. For each function, identify the x- and y-intercepts, determine the equation of the axis of symmetry and the coordinates of the vertex, and state the domain and range.

a)



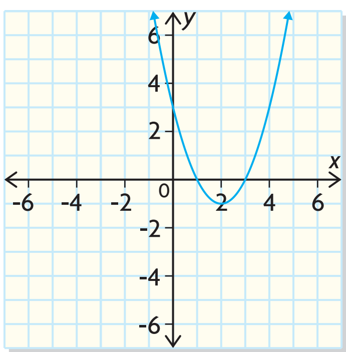
b)



PRACTISING

4. For each function, identify the equation of the axis of symmetry, determine the coordinates of the vertex, and state the domain and range.

a)



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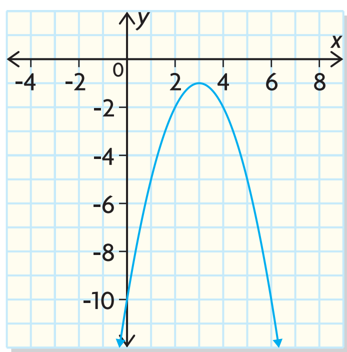

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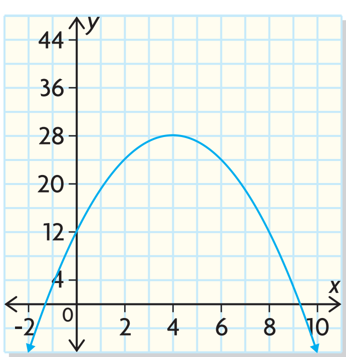

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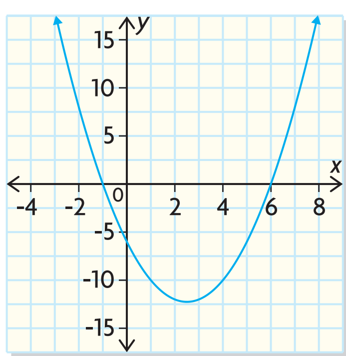

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5. Each parabola in question 4 is defined by one of the functions below.

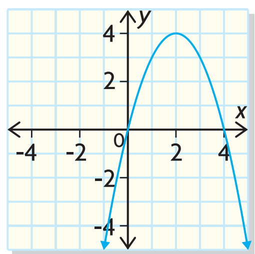
a) $f(x) = x^2 - 5x - 6$ **c)** $f(x) = -x^2 + 6x - 10$

b) $f(x) = -x^2 + 8x + 12$ **d)** $f(x) = x^2 - 4x + 3$

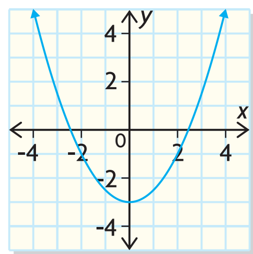
Identify the function that defines each graph. Then verify the coordinates of the vertex that you determined in question 4.

6. State whether each parabola has a minimum or maximum value, and then determine this value.

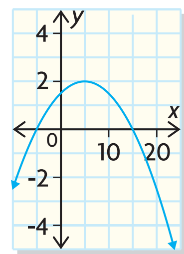
a)



b)



c)



x	-4	-2	0	2	4
y					

7. **a)** Complete the table of values shown for each of the following functions.

i) $y = -\frac{1}{2}x^2 + 5$ **ii)** $y = \frac{3}{2}x^2 - 2$

b) Graph the points in your table of values.

c) State the domain and range of the function.

8. **a)** Graph the functions $y = 2x^2$ and $y = -2x^2$.

b) How are the graphs the same? How are the graphs different?

c) Suppose that the graphs were modified so that they became the graphs of $y = 2x^2 + 4$ and $y = -2x^2 + 4$. Predict the vertex of each function, and explain your prediction.

9. For each of the following, both points, (x, y), are located on the same parabola. Determine the equation of the axis of symmetry for each parabola.

a) $(0, 2)$ and $(6, 2)$ **c)** $(-6, 0)$ and $(2, 0)$

b) $(1, -3)$ and $(9, -3)$ **d)** $(-5, -1)$ and $(3, -1)$

10. A parabola has x-intercepts $x = 3$ and $x = -9$. Determine the equation of the axis of symmetry for the parabola.

11. **a)** Graph each function.

i) $f(x) = 2x^2 + 3$ **iii)** $f(x) = x^2 - 6x + 9$

ii) $f(x) = -x^2 - 7x + 4$ **iv)** $f(x) = \frac{1}{2}x^2 - 4x + 3$

b) Determine the equation of the axis of symmetry and the coordinates of the vertex for each parabola.

c) State the domain and range of each function.

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12. In southern Alberta, near Fort Macleod, you will find the famous Head-Smashed-In Buffalo Jump. In a form of hunting, Blackfoot once herded buffalo and then stampeded the buffalo over the cliffs. If the height of a buffalo above the base of the cliff, $f(x)$, in metres, can be modelled by the function

$$f(x) = -4.9x^2 + 12$$

where x is the time in seconds after the buffalo jumped, how long was the buffalo in the air?

13. In the game of football, a team can score by kicking the ball over a bar and between two uprights. For a kick in a particular game, the height of the ball above the ground, y, in metres, can be modelled by the function

$$y = -4.9x^2 + 25x$$

where x is the time in seconds after the ball left the foot of the player.

a) Determine the maximum height that this kick reached, to the nearest tenth of a metre.
b) State any restrictions that the context imposes on the domain and range of the function.
c) How long was the ball in the air?

14. An annual fireworks festival, held near the seawall in downtown Vancouver, choreographs rocket launches to music. The height of one rocket, $h(t)$, in metres over time, t, in seconds, is modelled by the function

$$h(t) = -4.9t^2 + 80t$$

Determine the domain and range of the function that defines the height of this rocket, to the nearest tenth of a metre.

15. Melinda and Genevieve live in houses that are next to each other. Melinda lives in a two-storey house, and Genevieve lives in a bungalow. They like to throw a tennis ball to each other through their open windows. The height of a tennis ball thrown from Melinda to Genevieve, $f(x)$, in feet, over time, x, measured in seconds is modelled by the function

$$f(x) = -5x^2 + 6x + 12$$

What is the domain and range of this function if Genevieve catches the ball 4 ft above the ground? Draw a diagram to support your answer.

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16. Sid knows that the points $(-1, 41)$ and $(5, 41)$ lie on a parabola defined by the function

$$f(x) = 4x^2 - 16x + 21$$

a) Does $f(x)$ have a maximum value or a minimum value? Explain.

b) Determine, in two different ways, the coordinates of the vertex of the parabola.

Closing

17. a) Explain the relationship that must exist between two points on a parabola if the x-coordinates of the points can be used to determine the equation of the axis of symmetry for the parabola.

b) How can the equation of the axis of symmetry be used to determine the coordinates of the vertex of the parabola?

Extending

18. Gamez Inc. makes handheld video game players. Last year, accountants modelled the company's profit using the equation

$$P = -5x^2 + 60x - 135$$

This year, accountants used the equation

$$P = -7x^2 + 70x - 63$$

In both equations, P represents the profit, in hundreds of thousands of dollars, and x represents the number of game players sold, in hundreds of thousands. If the same number of game players were sold in these years, did Gamez Inc.'s profit increase? Justify your answer.

19. A parabola has a range of $\{y \mid y \leq 14.5, y \in \mathrm{R}\}$ and a y-intercept of 10. The axis of symmetry of the parabola includes point $(-3, 5)$. Write the function that defines the parabola in standard form if $a = \frac{-1}{2}$.

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6.3 Factored Form of a Quadratic Function

GOAL

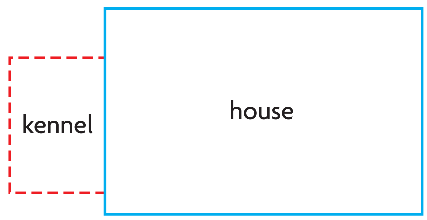
Relate the factors of a quadratic function to the characteristics of its graph.

YOU WILL NEED

- graph paper and ruler OR graphing technology

INVESTIGATE the Math

Ataneq takes tourists on dogsled rides. He needs to build a kennel to separate some of his dogs from the other dogs in his team. He has budgeted for 40 m of fence. He plans to place the kennel against part of his home, to save on materials.



EXPLORE…

- John has made a catapult to launch baseballs. John positions the catapult and then launches a ball. The height of the ball, $h(t)$, in metres, over time, t, in seconds, can be modelled by the function $h(t) = -4.9t^2 + 14.7t$ From what height did John launch the ball? How long was the ball in the air?

? What dimensions should Ataneq use to maximize the area of the kennel?

A. Using x to represent the width of the kennel, create an expression for the length of the kennel.

B. Write a function, in terms of x, that defines the area of the kennel. Identify the factors in your function.

C. Create a table of values for the function, and then graph it.

D. Does the function contain a maximum or a minimum value? Explain.

E. Determine the x-intercepts of the parabola.

F. Determine the equation of the axis of symmetry of the parabola and the coordinates of the vertex.

G. What are the dimensions that maximize the area of the kennel?

Reflecting

H. How are the x-intercepts of the parabola related to the factors of your function?

I. Explain why having a quadratic function in factored form is useful when graphing the parabola.

Communication *Tip*

A quadratic function is in factored form when it is written in the form $y = a(x - r)(x - s)$

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APPLY the Math

EXAMPLE 1 Graphing a quadratic function given in standard form

Sketch the graph of the quadratic function:

$$f(x) = 2x^2 + 14x + 12$$

State the domain and range of the function.

Arvin's Solution

$f(x) = 2x^2 + 14x + 12$
The coefficient of x^2 is $+2$, so the parabola opens upward.

The parabola opens upward when a is positive in the standard form of the function.

$f(x) = 2(x^2 + 7x + 6)$
$f(x) = 2(x + 1)(x + 6)$

I factored the expression on the right side so that I could determine the **zeros** of the function.

zero
In a function, a value of the variable that makes the value of the function equal to zero.

Zeros:
$0 = 2(x + 1)(x + 6)$
$x + 1 = 0$ or $x + 6 = 0$
$x = -1$ $\qquad$ $x = -6$
The x-intercepts are $x = -1$ and $x = -6$.

To determine the zeros, I set $f(x)$ equal to zero. I knew that a product is zero only when one or more of its factors are zero, so I set each factor equal to zero and solved each equation.

The values of x at the zeros of the function are also the x-intercepts.

y-intercept:
$f(0) = 2(0 + 1)(0 + 6)$
$f(0) = 2(1)(6)$
$f(0) = 12$
The y-intercept is 12.

I knew that the y-intercept is 12 from the standard form of the quadratic function. However, I decided to verify that my factoring was correct.

I noticed that this value can be obtained by multiplying the values of a, r, and s from the factored form of the function: $f(x) = a(x - r)(x - s)$

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Axis of symmetry:

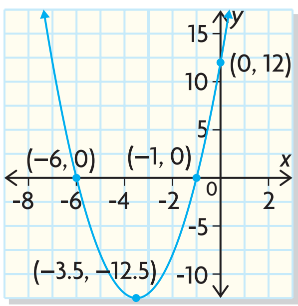
$$x = \frac{-6 + (-1)}{2}$$
$$x = -3.5$$

The axis of symmetry passes through the midpoint of the line segment that joins the x-intercepts. I calculated the mean of the two x-intercepts to determine the equation of the axis of symmetry.

$$f(x) = 2(x + 1)(x + 6)$$
$$f(-3.5) = 2(-3.5 + 1)(-3.5 + 6)$$
$$f(-3.5) = 2(-2.5)(2.5)$$
$$f(-3.5) = -12.5$$

The vertex of the parabola is $(-3.5, -12.5)$.

The vertex lies on the axis of symmetry, so its x-coordinate is -3.5. I substituted -3.5 into the equation to determine the y-coordinate of the vertex.



I plotted the x-intercepts, y-intercept, and vertex and then joined these points with a smooth curve.

Domain and range:
$\{(x, y) \mid x \in \mathrm{R}, y \geq -12.5, y \in \mathrm{R}\}$

The only restriction on the variables is that y must be greater than or equal to -12.5, the minimum value of the function.

Your Turn

Sketch the graph of the following function:

$$f(x) = -3x^2 + 6x - 3$$

a) How does the graph of this function differ from the graph in *Example 1*?

b) How are the x-intercepts related to the vertex? Explain.

EXAMPLE 2 Using a partial factoring strategy to sketch the graph of a quadratic function

Sketch the graph of the following quadratic function:

$$f(x) = -x^2 + 6x + 10$$

State the domain and range of the function.

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Elliot's Solution

$$f(x) = -x^2 + 6x + 10$$
$$f(x) = -x(x - 6) + 10$$

I couldn't identify two integers with a product of 10 and a sum of 6, so I couldn't factor the expression. I decided to remove a partial factor of $-x$ from the first two terms. I did this so that I could determine the x-coordinates of the points that have 10 as their y-coordinate.

$-x = 0 \qquad x - 6 = 0$

$x = 0 \qquad x = 6$

$f(0) = 10 \qquad f(6) = 10$

The points (0, 10) and (6, 10) belong to the given quadratic function.

I determined two points in the function by setting each partial factor equal to zero.

When either factor is zero, the product of the factors is zero, so the value of the function is 10.

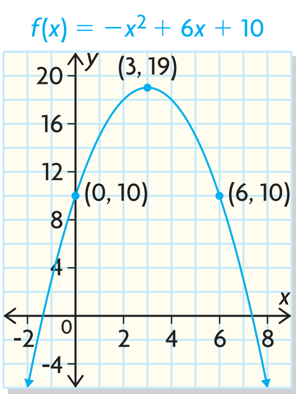
$$x = \frac{0 + 6}{2}$$
$$x = 3$$

Because (0, 10) and (6, 10) have the same y-coordinate, they are the same horizontal distance from the axis of symmetry. I determined the equation of the axis of symmetry by calculating the mean of the x-coordinates of these two points.

$$f(3) = -(3)^2 + 6(3) + 10$$
$$f(3) = -9 + 18 + 10$$
$$f(3) = 19$$

I determined the y-coordinate of the vertex.

The vertex is (3, 19).



The coefficient of the x^2 term is negative, so the parabola opens downward.

I used the vertex, as well as (0, 10) and (6, 10), to sketch the parabola.

Domain: $\{x \mid x \in \mathrm{R}\}$

Range: $\{y \mid y \leq 19, y \in \mathrm{R}\}$

The only restriction on the variables is that y must be less than or equal to 19, the maximum value of the function.

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Your Turn

a) i) Apply the partial factoring strategy to locate two points that have the same y-coordinate on the following function:

$$f(x) = -x^2 - 3x + 12$$

ii) Determine the axis of symmetry and the location of the vertex of the function from part i).

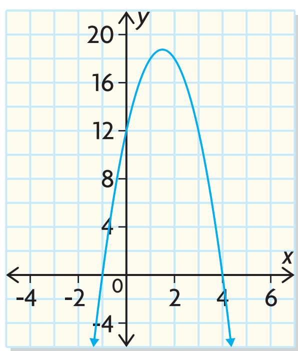
iii) Explain how the process you used in parts i) and ii) is different from factoring a quadratic function.

b) Explain whether you would use partial factoring to graph the function

$$g(x) = -x^2 - 4x + 12$$

EXAMPLE 3 Determining the equation of a quadratic function, given its graph

Determine the function that defines this parabola. Write the function in standard form.



Indira's Solution

The x-intercepts are $x = -1$ and $x = 4$. The zeros of the function occur when x has values of -1 and 4.

$y = a(x - r)(x - s)$

$y = a[x - (-1)][x - (4)]$

$y = a(x + 1)(x - 4)$

The graph is a parabola, so it is defined by a quadratic function.

I located the x-intercepts and used them to determine the zeros of the function. I wrote the factored form of the quadratic function, substituting -1 and 4 for r and s.

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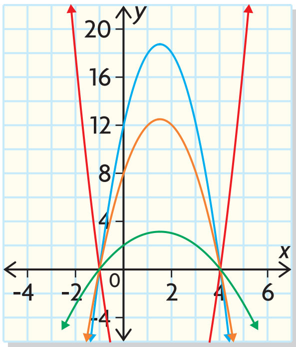


I knew that there are infinitely many quadratic functions that have these two zeros, depending on the value of *a*. I had to determine the value of *a* for the function that defines the blue graph.

The y-intercept is 12.

$$y = a(x + 1)(x - 4)$$
$$(12) = a[(0) + 1][(0) - 4]$$
$$12 = a(1)(-4)$$
$$12 = -4a$$
$$-3 = a$$

From the graph, I determined the coordinates of the *y*-intercept.

Because these coordinates are integers, I decided to use the *y*-intercept to solve for *a*.

In factored form, the quadratic function is

$$y = -3(x + 1)(x - 4)$$

I substituted the value of *a* into my equation.

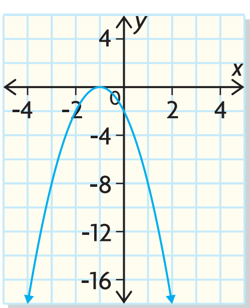
In standard form, the quadratic function is

$$y = -3(x^2 - 3x - 4)$$
$$y = -3x^2 + 9x + 12$$

My equation seems reasonable, because it defines a graph with a *y*-intercept of 12 and a parabola that opens downward.

Your Turn

If a parabola has only one x-intercept, how could you determine the quadratic function that defines it, written in factored form? Explain using the given graph.



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EXAMPLE 4 Solving a problem modelled by a quadratic function in factored form

The members of a Ukrainian church hold a fundraiser every Friday night in the summer. They usually charge \$6 for a plate of perogies. They know, from previous Fridays, that 120 plates of perogies can be sold at the \$6 price but, for each \$1 price increase, 10 fewer plates will be sold. What should the members charge if they want to raise as much money as they can for the church?

Krystina's Solution: Using the properties of the function

Let y represent the total revenue.
$y = (\text{Number of plates})(\text{Price})$
Let x represent the number of \$1 price increases.
$y = (120 - 10x)(6 + x)$

For each price increase, x, I knew that $10x$ fewer plates will be sold.

If I expanded the factors in my function, I would create an x^2 term. This means that the function I have defined is quadratic and its graph is a parabola.

$0 = (120 - 10x)(6 + x)$

$120 - 10x = 0$ or $6 + x = 0$

$-10x = -120$ $\qquad x = -6$

$x = 12$

The x-intercepts are $x = -6$ and $x = 12$.

To determine the zeros of the function, I substituted zero for y. A product is zero only when one or both of its factors are zero, so I set each factor equal to zero and solved each equation.

$$x = \frac{12 + (-6)}{2}$$
$$x = 3$$

$y = (120 - 10x)(6 + x)$
$y = [120 - 10(3)][6 + (3)]$
$y = (90)(9)$
$y = 810$

The coordinates of the vertex are (3, 810).

I determined the equation of the axis of symmetry for the parabola by calculating the mean distance between the x-intercepts.

I determined the y-coordinate of the vertex by substituting into my initial equation.

To generate as much revenue as possible, the members of the church should charge \$6 + \$3 or \$9 for a plate of perogies. This will provide revenue of \$810.

The vertex describes the maximum value of the function. Maximum sales of \$810 occur when the price is raised by \$3.

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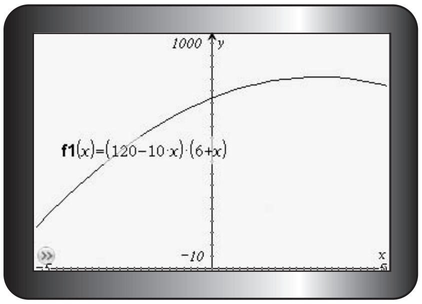
Jennifer's Solution: Using graphing technology

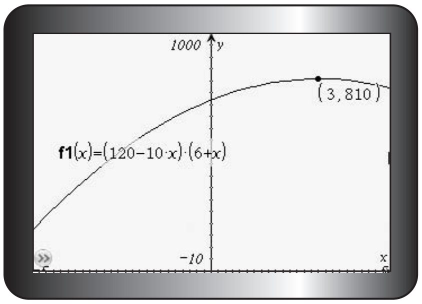
Revenue = (Number of plates)(Price)

$y = (120 - 10x)(6 + x)$

I let y represent Revenue and I let x represent the number of \$1 price increases. For each \$1 price increase, I knew that 10 fewer plates will be sold.



I graphed the equation on a calculator. Since a reduced price may result in maximum revenue, I set my domain to a minimum value of −5 and a maximum value of 5.



I used the calculator to locate the vertex of the parabola.

The members of the church should charge \$3 more than the current price of \$6 for a plate of perogies. If they charge \$9, they will reach the maximum revenue of \$810.

Your Turn

A career and technology class at a high school in Langley, British Columbia, operates a small T-shirt business out of the school. Over the last few years, the shop has had monthly sales of 300 T-shirts at a price of \$15 per T-shirt. The students have learned that for every \$2 increase in price, they will sell 20 fewer T-shirts each month. What should they charge for their T-shirts to maximize their monthly revenue?

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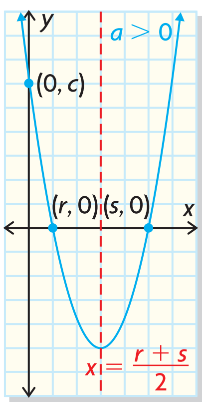
In Summary

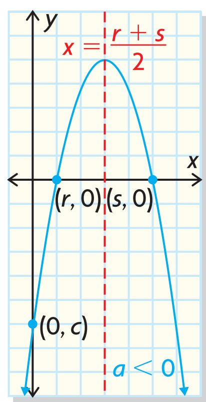
Key Ideas

- When a quadratic function is written in factored form
  $$y = a(x - r)(x - s)$$
  each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.
- The zeros of a quadratic function correspond to the x-intercepts of the parabola that is defined by the function.
- If a parabola has one or two x-intercepts, the equation of the parabola can be written in factored form using the x-intercept(s) and the coordinates of one other point on the parabola.
- Quadratic functions without any zeros cannot be written in factored form.

Need to Know

- A quadratic function that is written in the form
  $$f(x) = a(x - r)(x - s)$$
  has the following characteristics:
  - The x-intercepts of the graph of the function are $x = r$ and $x = s$.
  - The linear equation of the axis of symmetry is $x = \dfrac{r + s}{2}$.
  - The y-intercept, c, is $c = a \cdot r \cdot s$.





- If a quadratic function has only one x-intercept, the factored form can be written as follows:
  $$f(x) = a(x - r)(x - r)$$
  $$f(x) = a(x - r)^2$$

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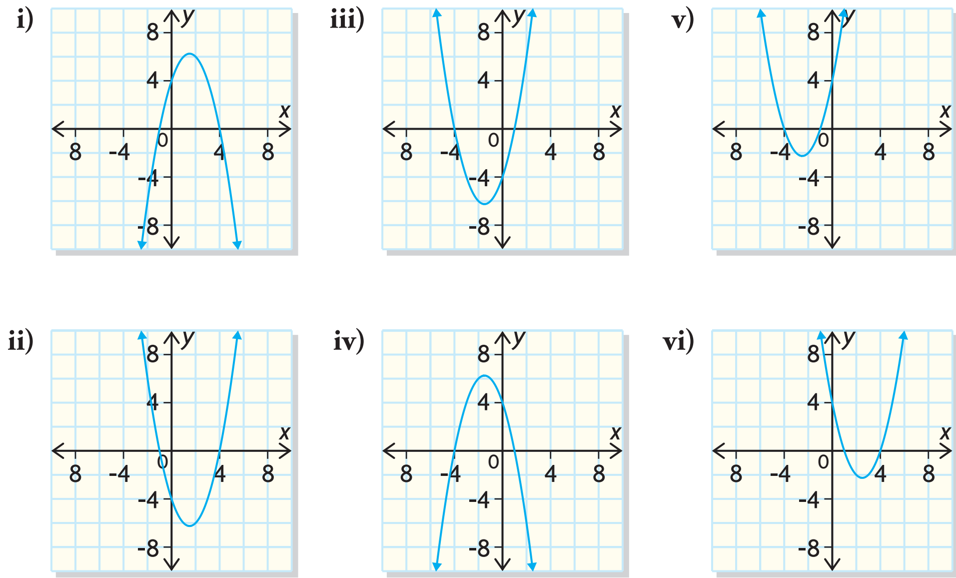
CHECK Your Understanding

1. Match each quadratic function with its corresponding parabola.

a) $f(x) = (x - 1)(x + 4)$ **d)** $f(x) = (x - 1)(x - 4)$
b) $f(x) = (x + 1)(x - 4)$ **e)** $f(x) = (1 - x)(x + 4)$
c) $f(x) = (x + 1)(x + 4)$ **f)** $f(x) = (x + 1)(4 - x)$



2. For each quadratic function below
i) determine the x-intercepts of the graph
ii) determine the y-intercept of the graph
iii) determine the equation of the axis of symmetry
iv) determine the coordinates of the vertex
v) sketch the graph

a) $f(x) = (x + 4)(x - 2)$ **c)** $h(x) = 2(x + 1)(x - 7)$
b) $g(x) = -2x(x - 3)$

3. A quadratic function has an equation that can be written in the form $f(x) = a(x - r)(x - s)$. The graph of the function has x-intercepts $x = -2$ and $x = 4$ and passes through point $(5, 7)$. Write the equation of the quadratic function.

PRACTISING

4. For each quadratic function, determine the x-intercepts, the y-intercept, the equation of the axis of symmetry, and the coordinates of the vertex of the graph.

a) $f(x) = (x - 1)(x + 1)$ **d)** $f(x) = -2(x - 2)(x + 1)$
b) $f(x) = (x + 2)(x + 2)$ **e)** $f(x) = 3(x - 2)^2$
c) $f(x) = (x - 3)(x - 3)$ **f)** $f(x) = 4(x - 1)^2$

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5. Sketch the graph of each function in question 4, and state the domain and range of the function.

6. Sketch the graph of

$$y = a(x - 3)(x + 1)$$

for $a = 3$. Describe how the graph would be different from your sketch if the value of a were 2, 1, 0, -1, -2, and -3.

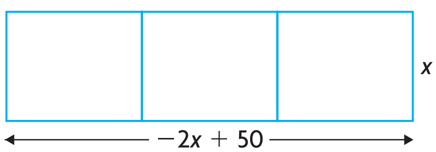
7. Sketch the graph of

$$y = (x - 3)(x + s)$$

for $s = 3$. Describe how the graph would be different from your sketch if the value of s were 2, 1, 0, -1, -2, and -3.8.

8. Byron is planning to build three attached rectangular enclosures for some of the animals on his farm. He bought 100 m of fencing. He wants to maximize the total area of the enclosures. He determined a function, $A(x)$, that models the total area in square metres, where x is the width of each rectangle:

$$A(x) = -2x^2 + 50x$$



a) Determine the maximum total area.
b) State the domain and range of the variables in the function.

9. Paulette owns a store that sells used video games in Red Deer, Alberta. She charges $10 for each used game. At this price, she sells 70 games a week. Experience has taught her that a $1 increase in the price results in five fewer games being sold per week. At what price should Paulette sell her games to maximize her sales? What will be her maximum revenue?

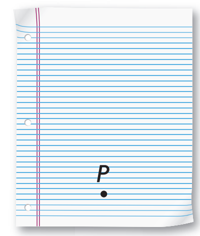
10. For each quadratic function below
i) use partial factoring to determine two points that are the same distance from the axis of symmetry
ii) determine the coordinates of the vertex
iii) sketch the graph

a) $f(x) = x^2 + 4x - 6$
b) $f(x) = x^2 - 8x + 13$
c) $f(x) = 2x^2 + 10x + 7$
d) $f(x) = -x^2 - 8x - 5$
e) $f(x) = -\frac{1}{2}x^2 + 2x - 3$
f) $f(x) = -2x^2 + 10x - 9$

Math in Action

Paper Parabolas

On an 8.5 in. by 11 in. piece of lined paper, mark a point that is close to the centre, near the bottom edge. Label this point *P*.



- Fold the paper, at any angle, so that any point on the bottom edge of the paper touches point *P*. Crease the paper along the fold line, and then open the paper.
- Fold the paper again, but at a different angle, so that another point on the bottom edge of the paper touches point *P*. Crease the paper.
- Continue this process until you have many different creases in the paper on both sides of *P*.
  - What shape emerges?
  - Compare your shape with the shapes made by other students. How are the shapes the same? How are they different?
  - How does changing the location of point *P* affect the shape that is formed?
  - The creases intersect at several points. How could you determine whether a set of these points is on a parabola?

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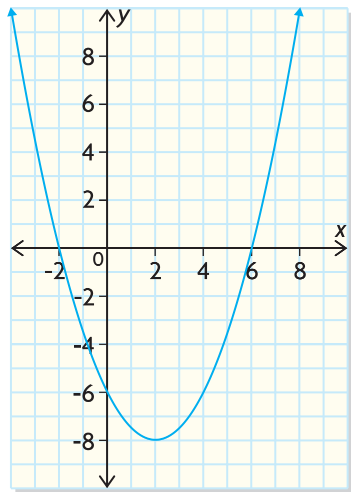
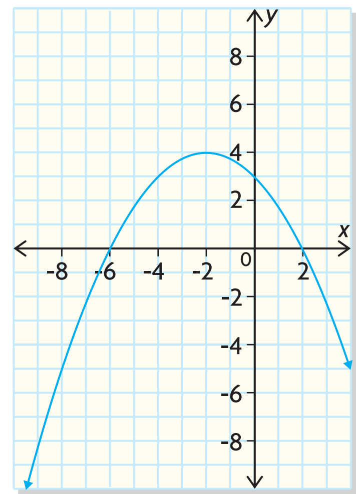
11. Determine the equation of the quadratic function that defines each parabola.

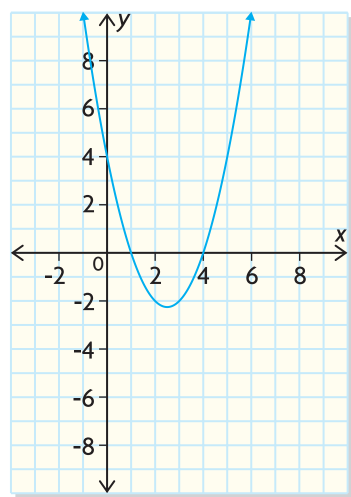
a)



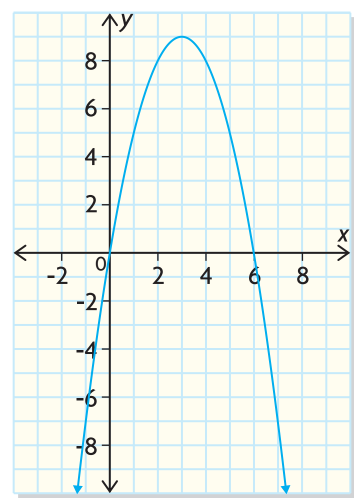
c)



b)



d)



12. a) Use two different algebraic strategies to determine the equation of the axis of symmetry and the vertex of the parabola defined by the following function:

$$f(x) = -2x^2 + 16x - 24$$

b) Which strategy do you prefer? Explain.

13. Determine the quadratic function that defines a parabola with x-intercepts $x = -1$ and $x = 3$ and y-intercept $y = -6$. Provide a sketch to support your work.

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14. How many zeros can a quadratic function have? Provide sketches to support your reasoning.

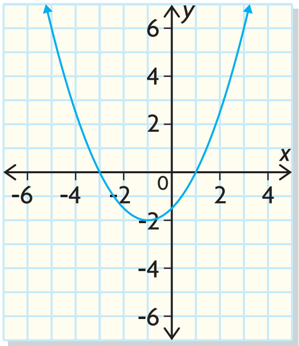
15. On the north side of Sir Winston Churchill Provincial Park, located near Lac La Biche, Alberta, people gather to witness the migration of American white pelicans. The pelicans dive underwater to catch fish. Someone observed that a pelican's depth underwater over time could be modelled by a parabola. One pelican was underwater for 4 s, and its maximum depth was 1 m.
    a) State the domain and range of the variables in this situation.
    b) Determine the quadratic function that defines the parabola.

16. Elizabeth wants to enclose the backyard of her house on three sides to form a rectangular play area for her children. She has decided to use one wall of the house and three sections of fence to create the enclosure. Elizabeth has budgeted \$800 for the fence. The fencing material she has chosen costs \$16/ft. Determine the dimensions that will provide Elizabeth with the largest play area.

17. A water rocket was launched from the ground, with an initial velocity of 32 m/s. The rocket achieved a height of 44 m after 2 s of flight. The rocket was in the air for 6 s.
    a) Determine the quadratic function that models the height of the rocket over time.
    b) State the domain and range of the variables.

Closing

18. Identify the key characteristics you could use to determine the quadratic function that defines this graph. Explain how you would use these characteristics.



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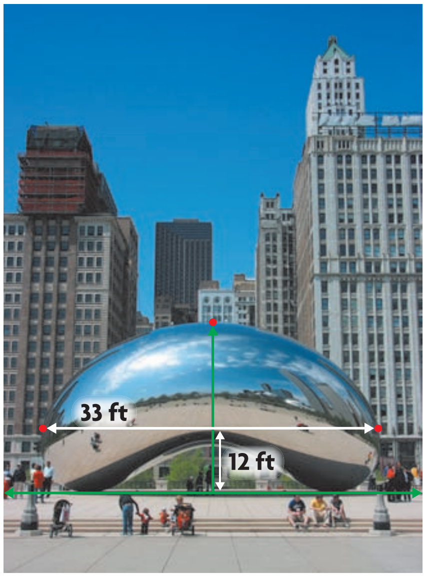
Extending



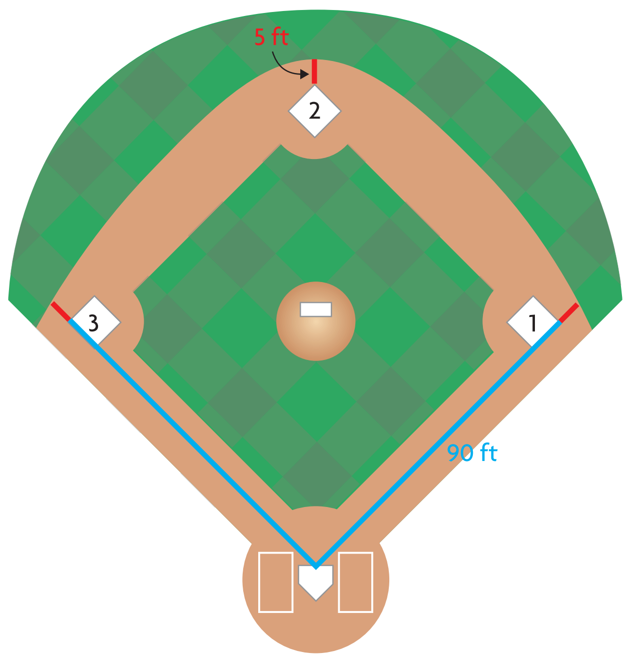
19. The Chicago Bean is a unique sculpture that was inspired by liquid mercury. It is 33 ft long and 33 ft high, with a 12 ft arch underneath. The top curved section that connects the three red dots in the photograph forms a parabola.

a) Determine the quadratic function that connects the three red dots. Assume that the ground (the green line) represents the x-axis of the graph. Write the function in standard form.

b) What is the domain and range of the variables?

c) If the parabola extended to the ground, what would the x-intercepts be, rounded to the nearest tenth?

20. A local baseball team has raised money to put new grass on the field. The curve where the infield ends can be modelled by a parabola. The foreman has marked out the key locations on the field, as shown in the diagram.



a) Determine a quadratic function that models the curve where the infield ends.

b) State the domain and range of the variables. Justify your decision.

c) Graph the quadratic function.

21. The National Basketball Association (NBA) mandates that every court must have the same dimensions. The length of the court must be 6 ft less than twice the width. The area of the court must be 4700 ft^2. Use this information to determine the dimensions of a basketball court used by the NBA.

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6 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What are the characteristics of a quadratic function?

A: The following are the key characteristics of a quadratic function:

- The equation is of degree 2.
- The graph is a parabola.
- The y-coordinate of the vertex of the parabola is a maximum if the parabola opens down and a minimum if the parabola opens up.
- The domain of the function is the set of real numbers. The range is restricted by the y-coordinate of the vertex. However, if the function is being used to model a situation, then the situation may restrict the domain and the range.
- The graph of the function contains a vertical axis of symmetry that passes through the vertex.

Study *Aid*
- See Lesson 6.1.
- Try Mid-Chapter Review Question 1.

Q: How can you use the information that is available from the standard or factored form of a quadratic function to sketch its graph?

A: The standard form is

$$y = ax^2 + bx + c$$

From this form, you can determine that the y-intercept of the graph is $y = c$.

The factored form is

$$y = a(x - r)(x - s)$$

From this form, you can determine

- the zeros (r and s), which provide the x-intercepts $x = r$ and $x = s$
- the y-intercept, determined by multiplying a, r, and s
- the equation of the axis of symmetry, $x = \frac{r + s}{2}$
- the location of the vertex, determined by substituting the x-coordinate of the vertex, $\frac{r + s}{2}$, into the equation

From both forms, you can determine the direction in which the parabola opens: upward when $a > 0$ and downward when $a < 0$.

Study *Aid*
- See Lesson 6.2, Examples 2 and 3, and Lesson 6.3, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 2 to 8 and 10.

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Study Aid

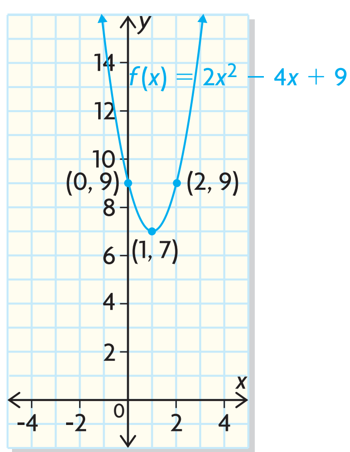
- See Lesson 6.3, Example 2.
- Try Mid-Chapter Review Question 9.

Q: What is partial factoring, and how is it used to sketch a graph?

A: Starting with the standard form of a quadratic function, factor only the terms that contain the variable x. The two partial factors can be used to locate two points that have the same y-coordinate, and so are equidistant from the axis of symmetry. Partial factoring can be used when a function cannot be factored completely.

For example: Sketch the graph of the following quadratic function:

$$f(x) = 2x^2 - 4x + 9$$

$f(x) = 2x^2 - 4x + 9$ $f(x) = 2x(x - 2) + 9$	Factor the terms that include x.
$2x = 0 \quad x - 2 = 0$ $x = 0 \quad x = 2$ (0, 9) and (2, 9) are points on the parabola.	Set each partial factor equal to zero. This allows you to identify two points on the graph with a y-coordinate of 9.
$x = \frac{0 + 2}{2}$ $x = 1$	Determine the equation of the axis of symmetry, which is located midway between (0, 9) and (2, 9).
$f(1) = 2(1)^2 - 4(1) + 9$ $f(1) = 7$ The vertex of the parabola is at (1, 7).	Locate the y-coordinate of the vertex by substituting into the quadratic function.
	From these three known points, you can sketch the graph.

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PRACTISING

Lesson 6.1

1. Which of the following are quadratic functions?

a) $y = 3x + 4$
b) $y = 2x(x - 5)$
c) $y = x^2 + 2x - 9$
d) $y = 2x^3 + 4x^2 - 5$

Lesson 6.2

2. a) Determine the y-intercept of the following quadratic function:

$$y = -x^2 + 8x$$

b) Graph the function.

c) From your graph, determine the equation of the axis of symmetry, the location of the vertex, the x-intercepts, and the domain and range of the function.

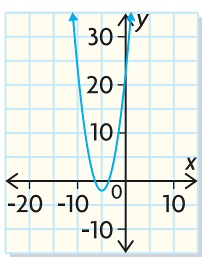
3. Consider the standard form of a quadratic function:

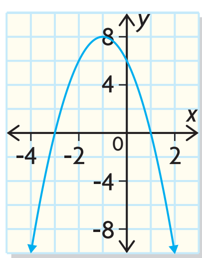
$$y = ax^2 + bx + c$$

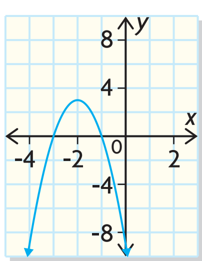
a) Explain how the value of a affects the graph of the parabola.

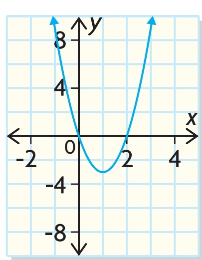
b) Provide supporting examples, with their graphs.

4. Match each graph to an equation below. Justify your decisions.

a)


c)


b)


d)


i) $y = -2x^2 - 4x + 6$
ii) $y = 3x^2 - 6x$
iii) $y = -3x^2 - 12x - 9$
iv) $y = x^2 + 10x + 23$

5. A flare is often used as a signal to attract rescue personnel in an emergency. When a flare is shot into the air, its height, $h(t)$, in metres, over time, t, in seconds can be modelled by

$$h(t) = -5t^2 + 120t$$

a) Identify the x-intercepts of the parabola.

b) When did the flare reach its maximum height, and what was this height?

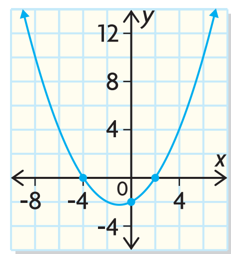
c) What was the height of the flare after 15 s?

d) State the domain and range of the function.

Lesson 6.3

6. The points $(-4, 6)$ and $(2, 6)$ lie on a parabola. Determine the equation of the axis of symmetry of the parabola.

7. Determine the equation of this quadratic function.



8. The zeros of a quadratic function are -6 and 12. The graph of the function intersects the y-axis at -36.

a) Determine the equation of the quadratic function.

b) Determine the coordinates of the vertex.

c) State the domain and range of the function.

9. Sketch the graph of the following quadratic function:

$$y = 3x^2 + 6x - 18$$

10. Pedalworks rents bicycles to tourists who want to explore the local trails. Data from previous rentals show that the shop will rent 7 more bicycles per day for every \$1.50 decrease in rental price. The shop currently rents 63 bicycles per day, at a rental price of \$39. How much should the shop charge to maximize revenue?

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6.4 Vertex Form of a Quadratic Function

YOU WILL NEED

- graph paper and ruler
- graphing technology
- calculator

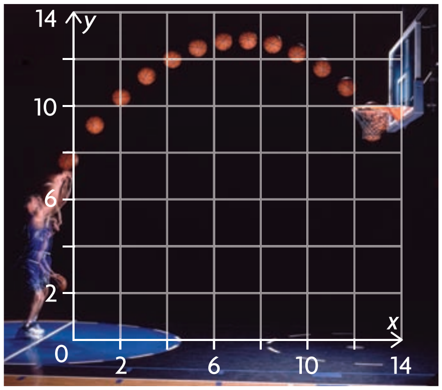
EXPLORE...

- Quadratic functions can be written in different forms. The basic quadratic function is $y = x^2$.
  Use a calculator to graph the following quadratic functions. Explain how the basic function is related to each function. Describe how the changes in the function affect the graph.
  a) $y = (x - 3)^2$
  b) $y = x^2 - 5$
  c) $y = (x + 1)^2 - 2$
  d) $y = (x + 4)^2 + 6$
  e) $y = -2(x + 1)^2 + 3$
  f) $y = 3(x - 2)^2 - 4$

GOAL

Graph a quadratic function in the form $y = a(x - h)^2 + k$, and relate the characteristics of the graph to its equation.

INVESTIGATE the Math



A high-school basketball coach brought in Judy, a trainer from one of the local college teams, to talk to the players about shot analysis. Judy demonstrated, using stroboscopic photographs, how shots can be analyzed and represented by quadratic functions. She used the following function to model a shot:

$$y = -0.1(x - 8)^2 + 13$$

In this function, x represents the horizontal distance, in feet, of the ball from the player and y represents the vertical height, in feet, of the ball above the floor.

Judy mentioned that once she had a quadratic equation in this form, she did not need the photographs. She could quickly sketch a graph of the path of the ball just by looking at the equation.

? How could Judy predict what the graph of the quadratic function would look like?

A. Graph the following function:

$$y = x^2$$

Change the graph by changing the coefficient of x^2. Try both positive and negative values. How do the parabolas change as you change this coefficient?

B. For each function you graphed in part A, determine the coordinates of the vertex and the equation of the axis of symmetry.

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C. Graph this function:

$$y = x^2 + 1$$

Change the graph by changing the constant. Try both positive and negative values. How do the parabolas change as you change the constant? How do the coordinates of the vertex and the equation of the axis of symmetry change?

D. Graph this function:

$$y = (x - 1)^2$$

Change the graph by changing the constant. Try both positive and negative values. How do the parabolas change as you change the constant? How do the coordinates of the vertex and the equation of the axis of symmetry change?

E. The equation that Judy used was expressed in vertex form:

$$y = a(x - h)^2 + k$$

Make a conjecture about how the values of a, h, and k determine the characteristics of a parabola.

Communication *Tip*

A quadratic function is in vertex form when it is written in the form $y = a(x - h)^2 + k$

F. Test your conjecture by predicting the characteristics of the graph of the following function:

$$y = -0.1(x - 8)^2 + 13$$

Use your predictions to sketch a graph of the function.

G. Using a graphing calculator, graph the function from part F:

$$y = -0.1(x - 8)^2 + 13$$

How does your sketch compare with this graph? Are your predictions supported? Explain.

Reflecting

H. Does the value of a in a quadratic function always represent the same characteristic of the parabola, whether the function is written in standard form, factored form, or vertex form? Explain.

I. Neil claims that when you are given the vertex form of a quadratic function, you can determine the domain and range without having to graph the function. Do you agree or disagree? Explain.

J. Which form of the quadratic function—standard, factored, or vertex—would you prefer to start with, if you wanted to sketch the graph of the function? Explain.

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APPLY the Math

EXAMPLE 1 Sketching the graph of a quadratic function given in vertex form

Sketch the graph of the following function:

$$f(x) = 2(x - 3)^2 - 4$$

State the domain and range of the function.

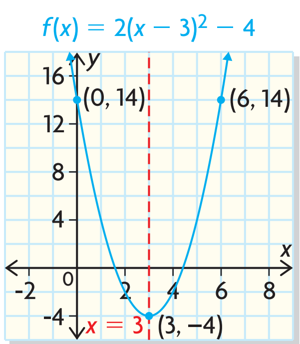
Samuel's Solution

$f(x) = 2(x - 3)^2 - 4$
Since $a > 0$, the parabola opens upward.
The vertex is at $(3, -4)$.
The equation of the axis of symmetry is $x = 3$.

The function was given in vertex form. I listed the characteristics of the parabola that I could determine from the equation.

$f(0) = 2(0 - 3)^2 - 4$
$f(0) = 2(-3)^2 - 4$
$f(0) = 2(9) - 4$
$f(0) = 18 - 4$
$f(0) = 14$
Point $(0, 14)$ is on the parabola.

To determine another point on the parabola, I substituted 0 for x.



I plotted the vertex and the point I had determined, (0, 14). Then I drew the axis of symmetry. I used symmetry to determine the point that is the same horizontal distance from (0, 14) to the axis of symmetry. This point is (6, 14). I connected all three points with a smooth curve.

Domain and range:
$\{(x, y) \mid x \in R, y \geq -4, y \in R\}$

Your Turn

Sketch the graph of the following function:

$$f(x) = -\frac{1}{2}(x + 6)^2 + 1$$

State the domain and range of the function. Justify your decision.

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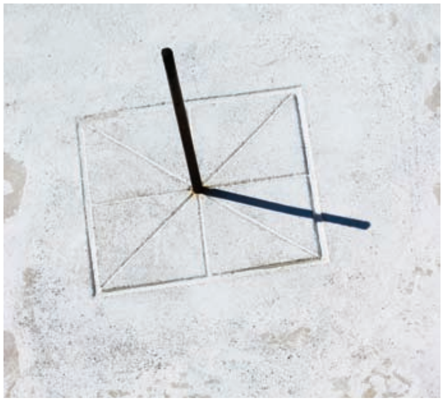
EXAMPLE 2 Determining the equation of a parabola using its graph

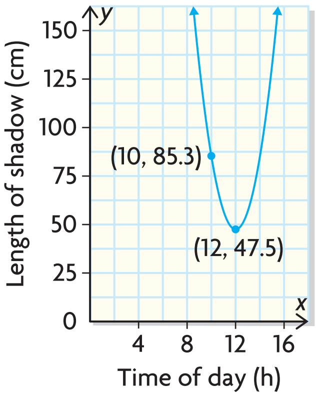


Liam measured the length of the shadow that was cast by a metre stick at 10 a.m. and at noon near his home in Saskatoon. Other students in his class also measured the shadow at different times during the day. They had read that, when graphed as shadow length versus time, the data should form a parabola with a minimum at noon, because the shadow is shortest at noon. Liam decided to try to predict the equation of the parabola, without the other students' data.

Determine the equation that represents the relationship between the time of day and the length of the shadow cast by a metre stick.

Liam's Solution

I have the points (10, 85.3) and (12, 47.5).



I measured the length of the shadow in centimetres. My measurements were 85.3 cm at 10 a.m. and 47.5 cm at noon.

I drew a sketch of a parabola using (12, 47.5) as the vertex, since the length of the shadow at noon should be the minimum value of the function.

$$f(x) = a(x - h)^2 + k$$
$$f(x) = a(x - 12)^2 + 47.5$$

I decided to use the vertex form of the quadratic function, since I already knew the values of h and k in this form.

Solving for a:

$$85.3 = a(10 - 12)^2 + 47.5$$
$$85.3 = a(-2)^2 + 47.5$$
$$85.3 = 4a + 47.5$$
$$37.8 = 4a$$
$$9.45 = a$$

I knew that (10, 85.3) is a point on the parabola. I substituted the coordinates of this point into the equation and then solved for a.

The function that represents the parabola is $f(x) = 9.45(x - 12)^2 + 47.5$

The domain and range of this function depend on the hours of daylight, which depends on the time of year.

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Your Turn

Donald, a classmate of Liam's, lives across the city. Donald measured the length of the shadow cast by a metre stick as 47.0 cm at noon and 198.2 cm at 4:00 p.m. Determine a quadratic function using Donald's data, and explain how his function is related to Liam's function.

EXAMPLE 3 Reasoning about the number of zeros that a quadratic function will have

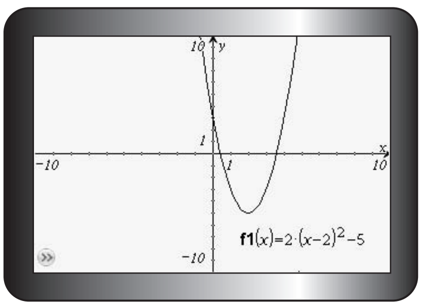
Randy claims that he can predict whether a quadratic function will have zero, one, or two zeros if the function is expressed in vertex form. How can you show that he is correct?

Eugene's Solution

$f(x) = 2(x - 2)^2 - 5$

Conjecture: two zeros

The vertex of the parabola that is defined by the function is at $(2, -5)$, so the vertex is below the x-axis. The parabola must open upward because a is positive. Therefore, I should observe two x-intercepts when I graph the function.



To test my conjecture, I graphed the function on a calculator. I can see two x-intercepts on my graph, so the function has two zeros.

The graph supports my conjecture.

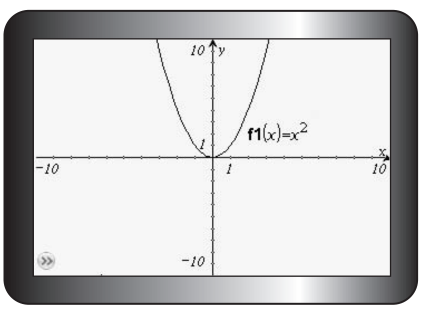
$f(x) = x^2$

$f(x) = (x - 0)^2 + 0$

Conjecture: one zero

I decided to use the basic quadratic function, since this provided me with a convenient location for the vertex, (0, 0).

Since the vertex is on the x-axis and the parabola opens up, this means that I should observe only one x-intercept when I graph the function.



To test my conjecture, I graphed the function on a calculator. Based on my graph, I concluded that the function has only one zero.

The graph supports my conjecture.

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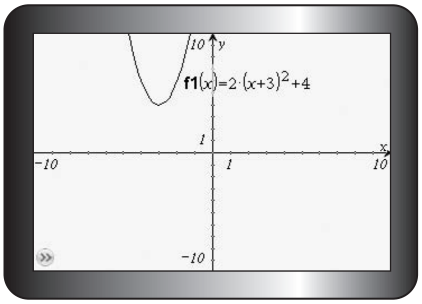
$f(x) = 2(x + 3)^2 + 4$

Conjecture: no zeros

The vertex of the parabola that is defined by this function is at $(-3, 4)$, and the parabola opens upward. The vertex lies above the x-axis, so I should observe no x-intercepts when I graph the function.



To test my conjecture, I graphed the function on a calculator. I concluded that the function has no zeros.

The graph supports my conjecture.

Your Turn

a) Define three different quadratic functions, in vertex form, that open downward. One function should have two zeros, another should have one zero, and the third should have no zeros.

b) Explain how you were able to connect the number of zeros to each function.

EXAMPLE 4 Solving a problem that can be modelled by a quadratic function

A soccer ball is kicked from the ground. After 2 s, the ball reaches its maximum height of 20 m. It lands on the ground at 4 s.

a) Determine the quadratic function that models the height of the kick.

b) Determine any restrictions that must be placed on the domain and range of the function.

c) What was the height of the ball at 1 s? When was the ball at the same height on the way down?

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Tia's Solution

a) Let x represent the elapsed time in seconds, and let y represent the height in metres.
$y = a(x - h)^2 + k$

Since I knew the maximum height and when it occurred, I also knew the coordinates of the vertex. I decided to use the vertex form to determine the equation.

The maximum height is 20 m at the elapsed time of 2 s.
Vertex:
$(x, y) = (2, 20)$

$y = a(x - 2)^2 + 20$

I substituted the known values.

Solving for a:

$$f(x) = a(4 - 2)^2 + 20$$
$$0 = a(2)^2 + 20$$
$$0 = 4a + 20$$
$$-20 = 4a$$
$$-5 = a$$

To determine the value of a, I substituted the coordinates of the point that corresponds to the ball hitting the ground, (4, 0).

The following quadratic function models the height of the kick:
$f(x) = -5(x - 2)^2 + 20$

b) Time at beginning of kick:
$x = 0$

Time when ball hits ground:
$x = 4$

Domain: $\{x \mid 0 \le x \le 4, x \in \mathrm{R}\}$

At the beginning of the kick, the time is 0 *s*. When the ball lands, the time is 4 *s*. I can only use x-values in this interval. Time in seconds is continuous, so the set is real numbers.

Vertex: (2, 20)
Height of ball at beginning of kick: 0 m
Height of ball at vertex: 20 m

Range: $\{y \mid 0 \le y \le 20, y \in \mathrm{R}\}$

The ball starts on the ground, at a height of 0 m, and rises to its greatest height, 20 m. The ball is not below the ground at any point. Height in metres is continuous, so the set is real numbers.

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c) $f(x) = -5(x - 2)^2 + 20$
$f(1) = -5(1 - 2)^2 + 20$
$f(1) = -5(-1)^2 + 20$
$f(1) = -5 + 20$
$f(1) = 15$

I used the vertex form of the quadratic function to determine the height of the ball at 1 s.

The ball was at a height of 15 m after 1 s. This occurred as the ball was rising.

Equation of the axis of symmetry:
$x = 2$

Symmetry provides the point (3, 15). The ball was also 15 m above the ground at 3 s.
This occurred as the ball was on its way down.

I knew that point (1, 15) is 1 unit to the left of the axis of symmetry of the parabola. The other point on the parabola, with height 15 m, should be 1 unit to the right of the axis of symmetry. This means that the *x*-coordinate of the point must be 3.

Your Turn

The goalkeeper kicked the soccer ball from the ground. It reached its maximum height of 24.2 m after 2.2 s. The ball was in the air for 4.4 s.

a) Define the quadratic function that models the height of the ball above the ground.
b) How is the equation for this function similar to the equation that Tia determined? Explain.
c) After 4 s, how high was the ball above the ground?

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In Summary

Key Idea

- The vertex form of the equation of a quadratic function is written as follows:

$$y = a(x - h)^2 + k$$

  The graph of the function can be sketched more easily using this form.

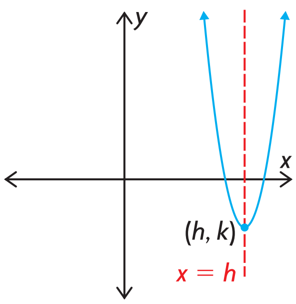
Need to Know

- A quadratic function that is written in vertex form,

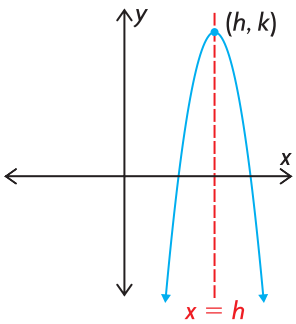
$$y = a(x - h)^2 + k$$

  has the following characteristics:
  – The vertex of the parabola has the coordinates (h, k).
  – The equation of the axis of symmetry of the parabola is $x = h$.
  – The parabola opens upward when $a > 0$, and the function has a minimum value of k when $x = h$.
  – The parabola opens downward when $a < 0$, and the function has a maximum value of k when $x = h$.

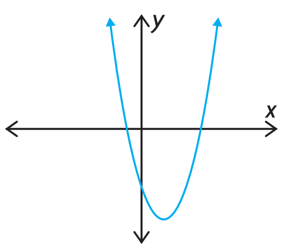
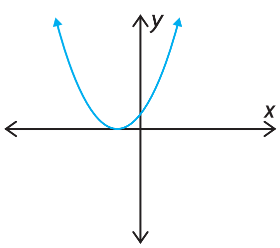
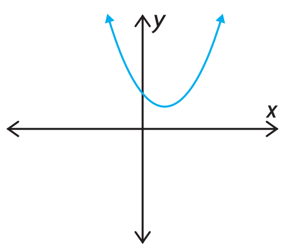
$y = a(x - h)^2 + k$
$a > 0$



$y = a(x - h)^2 + k$
$a < 0$



- A parabola may have zero, one, or two x-intercepts, depending on the location of the vertex and the direction in which the parabola opens. By examining the vertex form of the quadratic function, it is possible to determine the number of zeros, and therefore the number of x-intercepts.

Two x-intercepts	One x-intercept	No x-intercepts
		

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CHECK Your Understanding

1. For each quadratic function below, identify the following:
i) the direction in which the parabola opens
ii) the coordinates of the vertex
iii) the equation of the axis of symmetry

a) $f(x) = (x - 3)^2 + 7$
b) $m(x) = -2(x + 7)^2 - 3$
c) $g(x) = 7(x - 2)^2 - 9$
d) $n(x) = \frac{1}{2}(x + 1)^2 + 10$
e) $r(x) = -2x^2 + 5$

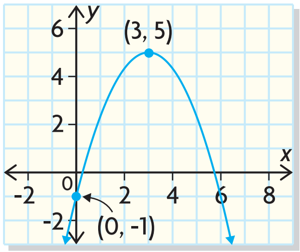
2. Predict which of the following functions have a maximum value and which have a minimum value. Also predict the number of x-intercepts that each function has. Test your predictions by sketching the graph of each function.
a) $f(x) = -x^2 + 3$
b) $q(x) = -(x + 2)^2 - 5$
c) $m(x) = (x + 4)^2 + 2$
d) $n(x) = (x - 3)^2 - 6$
e) $r(x) = 2(x - 4)^2 + 2$

3. Determine the value of a, if point $(-1, 4)$ is on the quadratic function:

$$f(x) = a(x + 2)^2 + 7$$

PRACTISING

4. Which equation represents the graph? Justify your decision.



A. $y = -\frac{2}{3}x^2 + 5$

B. $y = -(x - 3)^2 + 5$

C. $y = -\frac{2}{3}(x - 3)^2 + 5$

D. $y = \frac{2}{3}(x - 3)^2 + 5$

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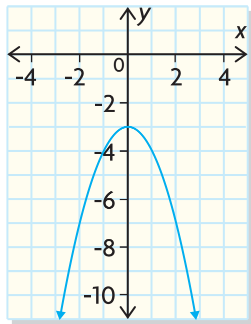
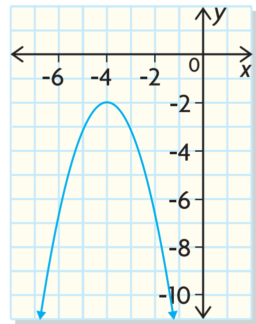
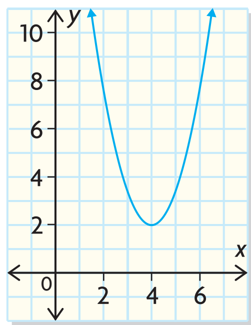
5. Match each equation with its corresponding graph. Explain your reasoning.

a) $y = (x - 3)^2$
b) $y = -(x + 4)^2 - 2$
c) $y = -x^2 - 3$
d) $y = (x - 4)^2 + 2$

i)


iii)


ii)


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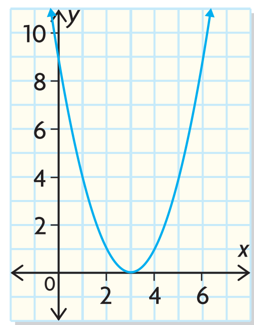
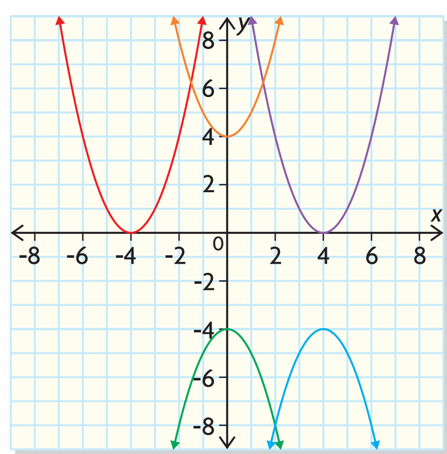


6. Explain how you would determine whether a parabola contains a minimum value or maximum value when the quadratic function that defines it is in vertex form:

$$y = a(x - h)^2 + k$$

Support your explanation with examples of functions and graphs.

7. State the equation of each function, if all the parabolas are congruent and if $a = 1$ or $a = -1$.



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8. Marleen and Candice are both 6 ft tall, and they play on the same university volleyball team. In a game, Candice set up Marleen with an outside high ball for an attack hit. Using a video of the game, their coach determined that the height of the ball above the court, in feet, on its path from Candice to Marleen could be defined by the function

$$h(x) = -0.03(x - 9)^2 + 8$$

where x is the horizontal distance, measured in feet, from one edge of the court.

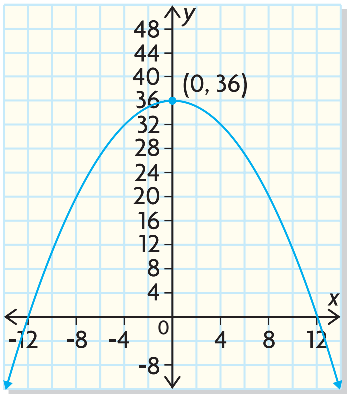
a) Determine the axis of symmetry of the parabola.
b) Marleen hit the ball at its highest point. How high above the court was the ball when she hit it?
c) How high was the ball when Candice set it, if she was 2 ft from the edge of the court?
d) State the range for the ball's path between Candice and Marleen. Justify your answer.

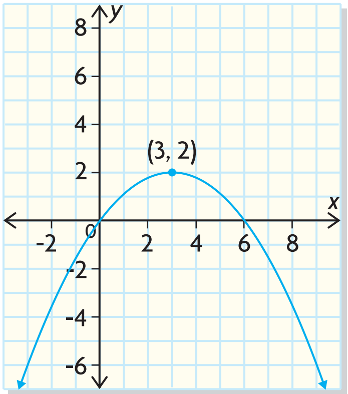
9. a) Write quadratic functions that define three different parabolas, all with their vertex at $(3, -1)$.
b) Predict how the graphs of the parabolas will be different from each other.
c) Graph each parabola on the same coordinate plane. How accurate were your predictions?

10. Without using a table of values or a graphing calculator, describe how you would graph the following function:

$$f(x) = 2(x - 1)^2 - 9$$

11. For each graph, determine the equation of the quadratic function in vertex form.

a)


b)


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12. The vertex of a parabola is at $(4, -12)$.
   a) Write a function to define all the parabolas with this vertex.
   b) A parabola with this vertex passes through point $(13, 15)$. Determine the function for the parabola.
   c) State the domain and range of the function you determined in part b).
   d) Graph the quadratic function you determined in part b).

13. The height of the water, $h(t)$, in metres, that is sprayed from a sprinkler at a local golf course, can be modelled by the function

$$h(t) = -4.9(t - 1.5)^2 + 11.3$$

   where time, t, is measured in seconds.
   a) Graph the function, and estimate the zeros of the function.
   b) What do the zeros represent in this situation?

14. A parabolic arch has x-intercepts $x = -6$ and $x = -1$. The parabola has a maximum height of 15 m.
   a) Determine the quadratic function that models the parabola.
   b) State the domain and range of the function.

15. Serge and a friend are throwing a paper airplane to each other. They stand 5 m apart from each other and catch the airplane at a height of 1 m above the ground. Serge throws the airplane on a parabolic flight path that achieves a minimum height of 0.5 m halfway to his friend.
   a) Determine a quadratic function that models the flight path for the height of the airplane.
   b) Determine the height of the plane when it is a horizontal distance of 1 m from Serge's friend.
   c) State the domain and range of the function.

Closing

16. Liz claims that she can sketch an accurate graph more easily if a quadratic function is given in vertex form, rather than in standard or factored form. Do you agree or disagree? Explain.

An atlatl is used to launch a dart. It has been used as a hunting tool by peoples all over the world for thousands of years.

Extending

17. Peter is studying the flight path of an atlatl dart for a physics project. In a trial toss on the sports field, Peter threw his dart 80 yd and hit a platform that was 2 yd above the ground. The maximum vertical height of the atlatl was 10 yd. The dart was 2 yd above the ground when released.
   a) Sketch a graph that models the flight path of the dart thrown by Peter.
   b) How far from Peter was the atlatl dart when it reached a vertical height of 8 yd? Explain.

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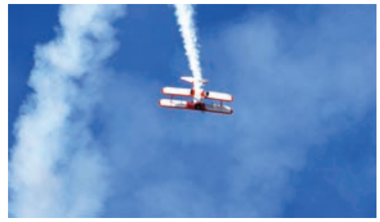
18. When an airplane is accelerated downward by combining its engine power with gravity, the airplane is said to be in a power dive. At the Abbotsford International Air Show, one of the stunt planes began such a manoeuvre. Selected data from the plane's flight log is shown below.



t	0	4	8	16
$h(t)$	520	200	40	200

a) Define a function, $h(t)$, that models the height of the plane above the ground, in metres, over time, t, in seconds, after the manoeuvre began.
b) How low to the ground did the plane get on this manoeuvre?
c) How long did it take for the plane to return to its initial altitude?

19. A bridge is going to be constructed over a river, as shown below. The supporting arch of the bridge will form a parabola. At the point where the bridge is going to be constructed, the river is 20 m wide from bank to bank. The arch will be anchored on the ground, 4 m from the edge of the riverbank on each side. The maximum height of the arch can be between 18 m and 22 m above the surface of the water. Create two different quadratic functions that model the supporting arch. Include a labelled graph for each arch.

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6.5 Solving Problems Using Quadratic Function Models

YOU WILL NEED

- graphing technology

GOAL

Solve problems involving situations that can be modelled by quadratic functions.

EXPLORE...

- Consider the three figures below.

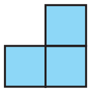

Figure 1

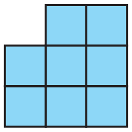

Figure 2

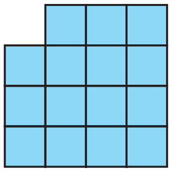

Figure 3

a) Create a table of values to compare the figure number, x, with the number of squares in each figure, y.
b) Graph the points. Then use your graph to predict the next three points in the pattern.
c) Determine a quadratic function that models the pattern.
d) Does the function you created enable you to state the domain and range for the context of the situation? Explain.

LEARN ABOUT the Math

The largest drop tower ride in the world is in an amusement park in Australia. Riders are lifted 119 m above the ground before being released. During the drop, the riders experience free fall for 3.8 s. The riders reach a speed of 37.5 m/s before magnetic brakes begin to slow down the platform. (This happens where the rail begins to move away from the supporting pillar in the photograph.)

A quadratic function can be used to model the height of the riders, H, in metres and the time, t, in seconds after the platform has been released.

The acceleration, a, is:

$$a = -0.5g$$

where g is the acceleration due to gravity. On Earth, $g = 9.8$ m/s^2.

? How high above the ground will the riders be when the brakes are applied?

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EXAMPLE 1 Applying quadratic models in problem solving

Determine the height above the ground where the brakes are applied.

Georgia's Solution

$H(t) = a(t - h)^2 + k$

I decided to use the vertex form of the quadratic function to model the height of the riders over time, since this problem contains information about the vertex.

The vertex is at (0, 119).
$h = 0$
$k = 119$
$H(t) = a(t - 0)^2 + 119$

The vertex represents the height of the platform, 119 m, at time 0, when the platform is released.

Acceleration:
$a = -0.5(9.8)$
$H(t) = -0.5(9.8)(t - 0)^2 + 119$
$H(t) = -4.9t^2 + 119$

The value of *a* is determined by gravity.

When $t = 3.8$ s:
$H(3.8) = -4.9(3.8)^2 + 119$
$H(3.8) = -4.9(14.44) + 119$
$H(3.8) = -70.756 + 119$
$H(3.8) = 48.244$

I substituted $t = 3.8$ into my function and evaluated the expression.

The riders are approximately 48 m above the ground when the magnetic brakes are applied, 3.8 s after the ride begins.

I decided to round my answer to the nearest whole number.

Reflecting

A. Explain why zero was used for the *t*-coordinate of the vertex.

B. Could the other forms of the quadratic function have been used in this situation? Explain.

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APPLY the Math

EXAMPLE 2 Representing a situation with a quadratic model

Mary sells sugar-coated mini-doughnuts at a carnival for \$6.00 a bag. Each day, she sells approximately 200 bags. Based on customer surveys, she knows that she will sell 20 more bags per day for each \$0.30 decrease in the price. What is the maximum daily revenue that Mary can achieve from doughnut sales, and what is the price per bag for this maximum revenue?

Pablo's Solution: Modelling using properties of the function

Let x represent the number of \$0.30 decreases in the price, where:
Revenue = (Price)(Number of bags sold)
Price = $6 - 0.30x$
Number of bags sold = $200 + 20x$

I defined a variable that connects the price per bag to the number of bags sold.

$$R = (6 - 0.30x)(200 + 20x)$$

If the price drops from \$6 in \$0.30 reductions x times, then the price per bag will be $(6 - 0.3x)$ and the number of bags sold will be $(200 + 20x)$.

$$0 = (6 - 0.3x)(200 + 20x)$$
$$0 = (6 - 0.3x) \quad \text{or} \quad 0 = (200 + 20x)$$

The equation is in factored form. I determined the zeros of the function.

$$0.3x = 6 \quad \text{or} \quad -20x = 200$$
$$x = \frac{6}{0.3} \quad \text{or} \quad x = \frac{200}{-20}$$
$$x = 20 \quad \text{or} \quad x = -10$$

Equation of axis of symmetry:
$$x = \frac{20 + (-10)}{2}$$
$$x = 5$$

Because the axis of symmetry passes through the vertex, which is the point of maximum revenue, I knew that 5 is the number of \$0.30 reductions that result in maximum revenue.

The price for a bag of doughnuts, P, that results in maximum revenue is
$$P = 6 - 0.30(5)$$
$$P = 6 - 1.50$$
$$P = 4.50$$
The price per bag that produces maximum revenue is \$4.50.

$$R = [6 - 0.3(5)]\,[200 + 20(5)]$$
$$R = [6 - 1.5]\,[200 + 100]$$
$$R = (4.5)(300)$$
$$R = 1350$$

The x-coordinate of the vertex is 5. I used this value to determine the y-coordinate of the vertex.

The maximum daily revenue for doughnut sales is \$1350, when the price per bag is \$4.50.

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Monique's Solution: Modelling using graphing technology

Let x represent the number of \$0.30 decreases in the price, where:
Revenue = (Price)(Number of bags sold)
Price $= 6 - 0.30x$
Number of bags sold $= 200 + 20x$

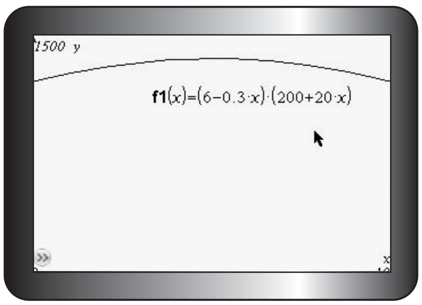
I defined a variable that connects the price per bag to the number of bags sold.

$$y = (6 - 0.30x)(200 + 20x)$$

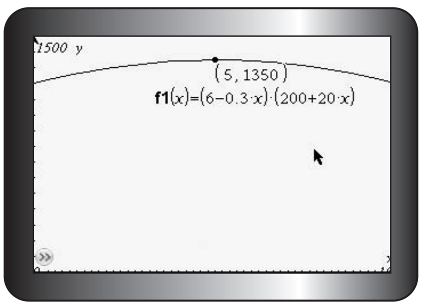
I created an equation that models the situation in the problem.

If the price drops by \$0.30 x times, then the price per bag will be $(6 - 0.3x)$. The number of bags sold will be $(200 + 20x)$.

I chose y to represent revenue.



I graphed the function on my calculator.



I used the calculator to determine the coordinates of the vertex.

The vertex is (5, 1350).

5 is the number of \$0.30 price reductions and \$1350 is the maximum daily revenue.

The price per bag of doughnuts, P, is
$P = 6 - 0.30(5)$
$P = 6 - 1.50$
$P = 4.50$

The price per bag that will produce the maximum daily revenue of \$1350 is \$4.50.

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Your Turn

Javier's saltwater candy booth is beside Mary's mini-doughnut stand at the carnival. Javier has determined that if he raises the price of his candy by \$0.25 per bag, he will sell 25 fewer bags each day. Javier currently sells 300 bags at \$5.50 per bag.

a) How much should Javier charge to generate the maximum revenue per day?
b) Describe any similarities and differences between the two booths.
c) What assumptions is Javier making if he is only looking at generating maximum revenue? Explain.

EXAMPLE 3 Solving a maximum problem with a quadratic function in standard form

A large cosmetics company is developing a new advertising campaign. The company has obtained data from the 2006 Census of Canada about the population of females between 25 and 65 years of age, the company's potential customers. Analysis of the data has provided the company with this quadratic model for the percent of women, y, at any given age, x, in the Canadian population:

$$y = -0.007x^2 + 0.623x - 5.094$$

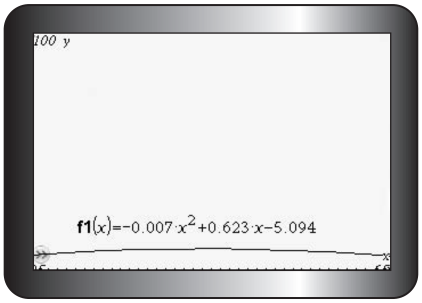
Determine the age group that the company should target to maximize the percent of the female population that uses its cosmetics.

Gina's Solution

Defined quadratic function:

$y = -0.007x^2 + 0.623x - 5.094$

I entered the function into my graphing calculator.



I used the information provided in the problem to set the domain as $25 \le x \le 65$.

I knew that the y-values in the question refer to a percent, so I set the range as $0 \le y \le 100$.

The result was a graph with a vertex that was difficult to see.

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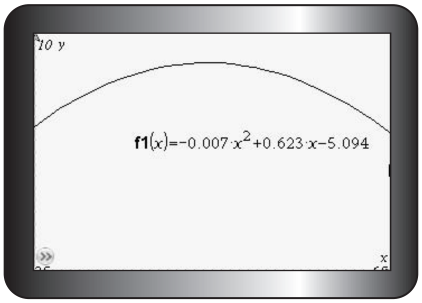
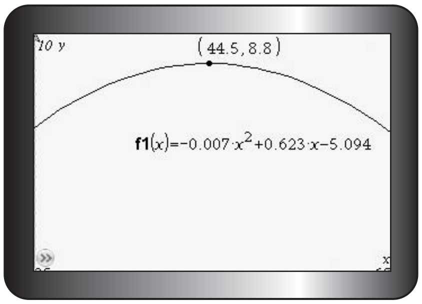


I adjusted the range so I could see the parabola more clearly.



I used the calculator to locate the coordinates of the vertex.

The vertex is at (44.5, 8.8).

44.5 is the age of women in Canada that results in the largest percent of the Canadian female population, which is about 9%.

The cosmetics company should target women around the age of 45, since this group makes up about 9% of women in Canada between the ages of 25 and 65.

I decided to round my answer to the nearest whole number.

Your Turn

The same company has a line of men's fragrances. Based on many mall surveys, the marketing team has determined that the function

$$y = -0.015x^2 + 0.981x - 6.828$$

models the percent of male respondents, y, at any given age, x, in the Canadian population.

a) What age should the marketing team target in its advertisements?

b) How is this model different from the model for women's cosmetics? How is it the same?

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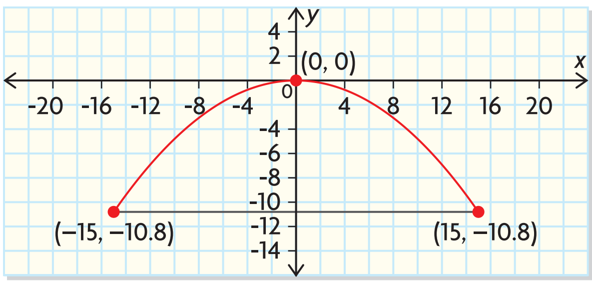
EXAMPLE 4 Determining a quadratic function that models a situation

The underside of a concrete underpass forms a parabolic arch. The arch is 30.0 m wide at the base and 10.8 m high in the centre. What would be the headroom at the edge of a sidewalk that starts 1.8 m from the base of the underpass? Would this amount of headroom be safe?

Domonika's Solution



To model the underpass, I decided to sketch a parabola with its vertex at the origin, (0, 0). I knew that this would simplify my calculations to determine points on the parabola.

I knew that the parabola must open downward to model the underpass.

The arch is 10.8 m high and 30 m wide, so I plotted points $(-15, -10.8)$ and $(15, -10.8)$.

I connected the three points with a smooth curve.

Using the vertex form of the quadratic function to model the parabola:

$$y = a(x - h)^2 + k$$
$$h = 0$$
$$k = 0$$

$$y = a(x - 0)^2 + 0$$
$$y = ax^2$$

$$-10.8 = a(15)^2$$
$$-\frac{10.8}{225} = a$$
$$-0.048 = a$$

I substituted the *x*- and *y*-coordinates of one of the known points into the quadratic function to determine the value of *a*.

$$y = -0.048x^2$$

I wrote the quadratic function that models the arch.

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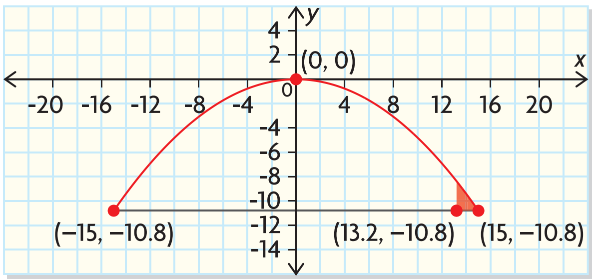
The edge of the sidewalk is 1.8 m from the base of the parabola.

$15.0 - 1.8 = 13.2$



I assumed that the sidewalk would be on the following line:

$$y = -10.8$$

I plotted point (13.2, −10.8).

I coloured the space between the sidewalk and the parabola. The highest point of the coloured region is the headroom above the edge of the sidewalk.

$y = -0.048(13.2)^2$
$y = -8.363\ldots$

I knew that the x-coordinate of the point directly above the edge of the sidewalk is 13.2. I substituted 13.2 into the quadratic function to determine the y-coordinate of this point.

Therefore, $(13.2, -8.4)$ is the approximate location of a point on the parabola that corresponds to the least headroom above the sidewalk.

Headroom above the sidewalk, H, is
$H = -8.363\ldots - (-10.8)$
$H = 2.436\ldots$ m

To determine the minimum headroom, I subtracted the y-coordinate of the base of the parabola from the y-coordinate of the point above the sidewalk.

On the edge of the sidewalk, the headroom will be about 2.4 m. This is higher than most people are tall, even when standing on a skateboard, so the plan for the sidewalk is safe.

Your Turn

Another concrete underpass, on the same stretch of road, is also 30.0 m wide at the base of its arch. However, this arch is 11.7 m high in the centre. The sidewalk under this arch will also be built so that it starts 1.8 m from the base of the underpass.

a) How much headroom will the sidewalk under this arch have?
b) What is the difference in headroom for the two sidewalks?

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In Summary

Key Idea

- The form of the quadratic function that you use to model a given situation depends on what you know about the relationship:
  - Use the vertex form when you know the vertex and an additional point on the parabola.
  - Use the factored form when you know the *x*-intercepts and an additional point on the parabola.

Need to Know

- When a function is a quadratic function, the maximum/minimum value corresponds to the *y*-coordinate of the vertex. The algebraic strategy you use to locate the vertex depends on the form of the quadratic function.

Quadratic Function	Algebraic Strategy to Determine the Vertex
Standard form: $f(x) = ax^2 + bx + c$	Use partial factoring to determine two points on the parabola with the same *y*-coordinate, then the axis of symmetry, and then the *y*-coordinate of the vertex: $f(x) = x(ax + b) + c$
Factored form: $f(x) = a(x - r)(x - s)$	Set each factor equal to zero to determine the zeros. Use the zeros to determine the equation of the axis of symmetry, then determine the *y*-coordinate of the vertex.
Vertex form: $f(x) = a(x - h)^2 + k$	The vertex is (h, k).

- All maximum/minimum problems can be solved using graphing technology, if you know the quadratic function that models the situation.

CHECK Your Understanding

1. Use the given information to express a quadratic function in vertex form:

$$y = a(x - h)^2 + k$$

a) $a = 3$, vertex at $(0, -5)$
b) $a = -1$, vertex at $(2, 3)$
c) $a = -3$, vertex at $(-1, 0)$
d) $a = 0.25$, vertex at $(-1.2, 4.8)$

2. The vertex of a parabola is at $(-5, 2)$. The parabola also includes point $(-1, -4)$. Determine the quadratic function that defines the parabola.

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PRACTISING

3. Determine the quadratic function, in vertex form, that defines each parabola.

a) vertex at $(0, 3)$, passes through $(-2, 11)$

b) vertex at $(3, 0)$, passes through $(-1, 8)$

c) vertex at $(-1, 4)$, passes through $(3, -12)$

d) vertex at $(2, -5)$, passes through $(4, -7)$

4. Determine the quadratic function that includes the factors $(x - 3)$ and $(x - 7)$ and the point $(6, 3)$. Write the equation for the function in vertex form.

5. For each quadratic function below

i) sketch the graph of the function

ii) state the maximum or minimum value

iii) express the function in standard form

a) $f(x) = -(x - 1)(x + 3)$

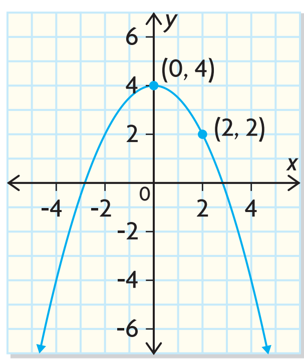
b) $g(x) = 4(x - 3)(x - 2)$

c) $m(x) = 3(x - 2)^2 - 4$

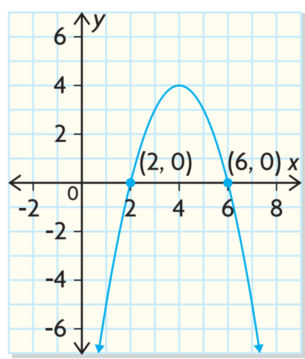
d) $n(x) = -0.25(x + 4)^2 + 5$

6. Determine the quadratic function, in vertex form, that defines each parabola.

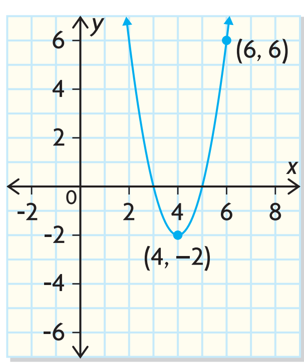
a)



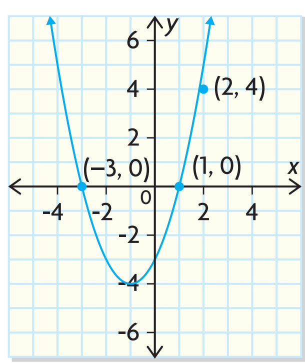
c)



b)



d)



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7. A stunt pilot performs a manoeuvre called the death spiral by flying the airplane straight up, cutting power to the engine, and then allowing the airplane to fall toward the ground before restarting the engine.
   a) The altimeter in the airplane reads 1375 m when the engine is turned off and reads 1000 m 15 s later. The airplane achieves its maximum height 5 s into the manoeuvre. Model the death spiral with a function, $h(t)$, that describes the height of the airplane, in metres, above the ground over the time, t, in seconds, after the engine is turned off.
   b) How high does the airplane get on this manoeuvre?

8. Bill and Ben are on a bridge, timing how long it takes stones they have dropped to hit the water below. A quadratic relation can be used to determine the distance, D, in metres, that a stone will fall in the time, t, in seconds after it is released. In this relation, $a = -0.5g$, where g is acceleration due to gravity, which is approximately 9.8 m/s^2 on Earth. Ben starts a timer when Bill releases a stone, and he stops the timer when the stone hits the water below. The mean time of several trials is 3 s.
   a) Determine a quadratic function that models the falling stones.
   b) How high above the water are Bill and Ben? Explain your answer.
   c) On Saturn's moon, Titan, the value of g is 1.35 m/s^2. Suppose that Bill and Ben are astronauts and they are standing at the top of a cliff on Titan. If they record the time for a stone to fall from their hands to the bottom of the cliff as 3 s, how high is the cliff?

9. A stone was thrown straight down from a high point on the Enderby Cliffs near Enderby, British Columbia. The stone travelled 180 m before hitting the ground. The stone took 5 s to reach the ground. The height of the stone, in metres, can be modelled by the quadratic function

$$h(t) = -5t^2 + V_o t + h_o$$

   where V_o represents the initial velocity of the rock, h_o represents the initial height, and t represents the time in seconds.
   a) Determine the initial velocity of the rock.
   b) Write the quadratic function from question 8a) in standard form. Provide your reasoning.

10. An ice cream shop sells 700 ice cream cones per day at a price of \$4.50. Based on the previous year's sales, the owners know that they will sell another 100 ice cream cones for every \$0.50 decrease in price.
   a) How should the owners price their cones if they want to maximize their daily revenue?
   b) How much extra revenue will be gained by the price change?

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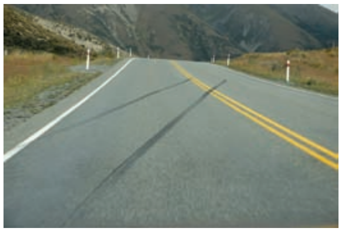
11. Police can use a quadratic function to model the relationship between the speed of a car and the length of the skid marks it makes on the pavement after the driver starts to brake. Based on this relationship, if a car is travelling at 100 km/h, the car will leave skid marks that are 70 m long under full braking.



a) Explain why the vertex of the function should be at (0, 0).

b) Determine a quadratic function that relates the speed of a car, x, measured in kilometres per hour, to the stopping distance, D, measured in metres.

c) How long would the skid marks be for a car that is travelling at 60 km/h, rounded to the nearest tenth?

d) What factors, other than speed, might affect the length of the skid marks? Explain.

12. The Lions Gate Bridge in Vancouver is a suspension bridge. The main span, between the two towers, is 472 m long. Large cables are attached to the top of both towers, 50 m above the road. Each large cable forms a parabola. The road is suspended from the large cables by a series of shorter vertical cables. The shortest vertical cable measures about 2 m. Use this information to determine a quadratic function that models one of the large cables.

13. A parking lot is being constructed under a historic hotel. The entrance to the parking lot is through a parabolic arch that is 9 ft high at its centre and 13 ft wide at its base.

a) Determine a quadratic function, $h(x)$, that relates the height of the arch, in feet, to its width, x, in feet. State the domain of this function.

b) What is the widest vehicle, 6 ft high, that can enter the parking lot through the arch?

c) What is the tallest vehicle, 6 ft wide, that can enter the parking lot through the arch?

d) Suggest wording for a sign to warn drivers about the restrictions for vehicles about to enter the parking lot.

14. A large radio-telescope dish was built in a hollow at Arecibo in Puerto Rico. The parabolic surface contains 38 778 slightly curved aluminum panels, each measuring 1 m wide and 2 m long. If a cross-section of the dish were placed on a coordinate grid so that the maximum width, 305 m, was aligned along the x-axis, the vertex of the parabola would be 21.94 m below the origin. Determine a quadratic function that models the cross-section of the dish.

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15. The owner of a small clothing company wants to create a mathematical model for the company's daily profit, p, in dollars, based on the selling price, d, in dollars, of the dresses made. The owner has noticed that the maximum daily profit the company has made is \$1600. This occurred when the dresses were sold for \$75 each. The owner also noticed that selling the dresses for \$50 resulted in a profit of \$1225. Use a quadratic function to model this company's daily profit.

Closing

16. A dinner theatre has 600 season ticket holders. The owners of the theatre have decided to raise the price of a season ticket from the current price of \$400. According to a recent survey of season ticket holders, for every \$50 increase in the price, 30 season ticket holders will not renew their seats.

a) What should the owners charge for each season ticket in order to maximize their revenue?

b) How many people will still buy season tickets if the owners decide to apply the new price?

c) How might the size of the audience affect the dinner theatre? Explain.

Extending

17. An oval reflecting pool has water fountains along its sides. Looking from one end of the pool, streams from fountains on each side of the pool cross each other over the middle of the pool at a height of 3 m. The two fountains are 10 m apart. Each fountain sprays an identical parabolic-shaped stream of water a total horizontal distance of 8 m toward the opposite side.

a) Determine an equation that models a stream of water from the left side and another equation that models a stream of water from the right side. Graph both equations on the same set of axes.

b) Determine the maximum height of the water.

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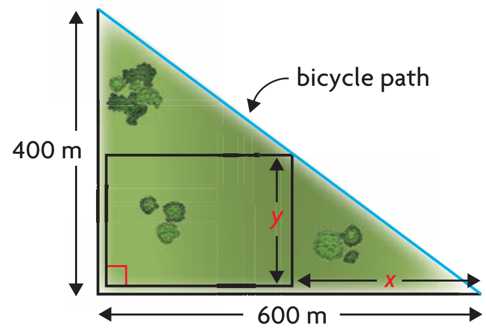
18. A straight bicycle path crosses two roads, forming a triangle as shown. A developer wants to create a rectangular section within the triangle and then subdivide the section into smaller rectangular lots. She wants the rectangle to be the maximum possible area.

The length of the rectangle can be defined as $(600 - x)$. The width of the rectangle, y, can be defined as $\frac{2x}{3}$.



a) Write a quadratic function that defines the area of the rectangular section of land.

b) What is the maximum area for the rectangle?

c) Determine the dimensions of the rectangle that result in the maximum area.

d) The developer plans to mark out 20 m by 30 m lots. How many lots will be available?

History Connection

Trap Shooting

Trap shooting involves hitting clay discs using a shotgun. The sport was introduced to the Olympics in 1900, but the first Olympic competition using the current rules was in 1952. In this competition, there are five shooting stations, and the shooters rotate through the stations. Targets are launched from a single "house." Programmed machines randomly launch the targets from the house in the proportion 10 to the right, 10 to the left, and 5 straight on, so the targets are similar and fair for all the shooters.

The shooters must hit as many targets as they can. Each shooter has 10 s in which to signal the release of a target. Once a shooter has fired a shot, the next shooter calls for a target. Men must fire 125 shots, and women must fire 75 shots.

Canadians have won gold medals twice in this sport. Walter Ewing won in 1908, and George Généreux won in 1952.

Susan Nattress is one of Canada's most decorated trap shooters. She was born in Medicine Hat, Alberta, and has competed in six Olympic Games.

A. Is a quadratic function an appropriate model to represent a "shot" in this competition? Explain.

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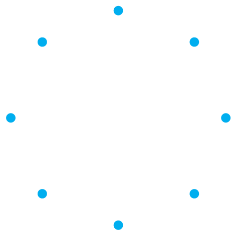
Applying Problem-Solving Strategies

Curious Counting Puzzles

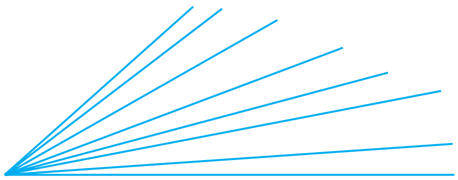
Consider the following puzzles:

How many handshakes will there be if 8 people must shake hands with each other only once?

How many lines can be drawn that connect each pair of points if there are 8 points in total?



How many acute angles are there in this diagram?



How are these three puzzles related to each other?

The Strategy

A. For each problem, use inductive reasoning to generate data. Begin your reasoning process by examining the cases where the number of people, points, and lines is 0. Then look at other cases by increasing the number of people, points, and lines systematically by 1 each time.

B. Graph your data. What type of function can be used to model each situation?

C. Determine the function that represents each relationship.

D. Explain how these problems are related to each other.

Creating a Variation of the Puzzle

E. Create a related problem that can be modelled by the function you determined above.

F. Explain how the function can be used to solve your problem.

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6 Chapter Self-Test

1. Sketch each of the following quadratic functions. Explain why you chose the method you used.
   a) $f(x) = x^2 - 8x + 12$
   b) $f(x) = -2(x + 1)(x - 5)$
   c) $f(x) = 0.5(x + 2)^2 - 7$
   d) $f(x) = -2x^2 - 8x$
2. Determine the y-intercept, x-intercepts, equation of the axis of symmetry, and vertex of the parabola that is defined by each quadratic function.
   a) $y = -1(x + 3)(x - 5)$
   b) $y = (2x - 3)(x + 4)$
3. Workers who were improving a section of highway near Rogers Pass, British Columbia, used dynamite to remove a rock obstruction. When the rock shattered, the height of one piece of rock, $h(t)$, in feet, could be modelled by the function
$$h(t) = -16t^2 + 160t$$
   where t represents the time, in seconds, after the blast.
   a) How long was the piece of rock in the air?
   b) How high was the piece of rock after 2 s?
   c) What was the maximum height of the piece of rock?
4. A quadratic function has zeros of -1 and -3, and includes the point (1, 24). Determine the quadratic function.
5. A parabola has a y-intercept of -4 and a vertex at $(3, -7)$. Determine the equation of the parabola in standard form.
6. Dimples the Clown has been charging \$260 to perform at a children's party. Dimples is too busy to keep up with his bookings, and thinks that charging more for his performances will result in fewer bookings but more revenue. If he raises the charge by \$80 per party, he expects to get one fewer booking per month. Dimples performs at 20 children's parties each month at his current price. How much should he charge to maximize his monthly revenue?
7. A builder wants to place a parabolic arch over the doorway of a home she is building. She wants the highest point in the arch to be over the middle of the doorway. The arch she wants to use is defined by the quadratic function
$$y = -\frac{5}{18}x^2 + 5x - \frac{25}{2}$$
   where x is the horizontal distance from the left edge of the building, in feet, and y is the height of the arch, in feet, above the steps. The doorway is 8 ft wide. What will be the height of the arch above the top step? Is this headroom reasonable? Explain.

WHAT DO You Think Now? Revisit **What Do You Think?** on page 321. How have your answers and explanations changed?

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6 Chapter Review

FREQUENTLY ASKED Questions

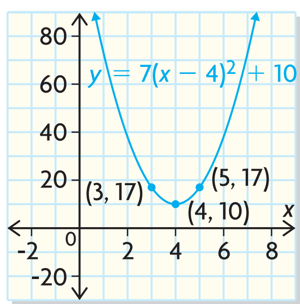
Study *Aid*

- See Lesson 6.4, Examples 1 and 2.
- Try Chapter Review Questions 10 and 11.

Q: How can you graph a quadratic function in vertex form, $y = a(x - h)^2 + k$?

A: Use the information provided by the form of the quadratic equation. For example: Sketch the graph of the following quadratic function:

$$y = 7(x - 4)^2 + 10$$

The vertex is at (4, 10).	Determine the coordinates of the vertex, (h, k).
$y = 7[(5) - 4]^2 + 10$ $y = 7(1)^2 + 10$ $y = 17$ One other point on the graph is (5, 17).	Locate one other point on the function by substituting a value for x into the equation. In this example, substitute 5 for x because the calculation is easy to check.
Another point on the graph is (3, 17), because 3 is the same distance from 4 as 5 is. (Another way of looking at this is that $(x - 4)^2 = 1$ when x is 5 and when x is 3.)	Apply symmetry to the first located point. In this example, the vertical line of symmetry is $x = 4$.
	Connect the three points with a smooth curve.

Study *Aid*

- See Lesson 6.4, Examples 2 and 4, and Lesson 6.5, Examples 1, 2, and 3.
- Try Chapter Review Questions 15 to 19.

Q: Which form of the quadratic function should you use to solve contextual problems?

A: If you want to know the maximum or minimum value, you should use the vertex form.
If you want to know the y-intercept, you should use the standard form.
If you want to know the x-intercepts, you should use the factored form.

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Q: How can you determine a quadratic function that models a situation described in a problem?

A1: If you are given the vertex and one other point on the parabola, write the function in whatever form is easiest to determine. You can then rewrite the function in standard form.

For example: The vertex of a parabola is $(-2, -4)$, and $(2, 6)$ is another point on the parabola.

$y = a(x - h)^2 + k$ $y = a(x + 2)^2 - 4$	Substitute (h, k), the coordinates of the vertex, into the vertex form of the quadratic function.
$y = a(x + 2)^2 - 4$ $(6) = a[(2) + 2]^2 - 4$ $6 = a[16] - 4$ $10 = 16a$ $\frac{10}{16} = a$ $\frac{5}{8} = a$	Using the coordinates of the other known point, (x, y), solve for a.
$y = \frac{5}{8}(x + 2)^2 - 4$	Substitute the value of a into your quadratic function.

A2: If you are given two x-intercepts and another point on the graph, substitute the coordinates of these points into the factored form of the quadratic function.

For example: The x-intercepts of a parabola are $x = -2$ and $x = 4$. Another point on the parabola is $(6, 64)$. Determine the quadratic function that defines the parabola.

$y = a(x - r)(x - s)$ $y = a(x + 2)(x - 4)$	Substitute $(r, 0)$ and $(s, 0)$ into the factored form of the quadratic function.
$y = a(x + 2)(x - 4)$ $(64) = a[(6) + 2][(6) - 4]$ $64 = a[16]$ $4 = a$	Substitute the coordinates, (x, y), of the other known point into your equation and solve for a.
$y = 4(x + 2)(x - 4)$	Substitute the value of a into your quadratic function.

Study | Aid

- See Lesson 6.4, Example 4, and Lesson 6.5, Examples 1 and 4.
- Try Chapter Review Questions 13 and 14.

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PRACTISING

Lesson 6.1

1. Graph the following quadratic functions without using technology.

a) $f(x) = x^2 - 6x + 8$

b) $g(x) = -2(x + 1)(x - 3)$

c) $h(x) = 0.5(x + 4)^2 - 2$

Lesson 6.2

2. Monish is taking a design course in high school. He wants to create a model of Winnipeg's River Arch digitally by placing one of the bases of this arch at the origin of a graph on a coordinate grid. He knows that the arch spans 23 m. Explain how to determine the equation of the axis of symmetry for his model.

3. The points $(-2, -41)$ and $(6, -41)$ are on the following quadratic function:

$$f(x) = -3x^2 + 12x - 5$$

Determine the vertex of the function.

4. Trap shooting is a sport in which a clay disk is launched into the air by a machine. Competitors are required to shoot the disk with a shotgun while the disk is in the air. The height, $h(t)$, in metres, of one clay disk after it is launched is modelled by the function

$$h(t) = -5t^2 + 30t + 2$$

where t represents time after launch, in seconds.

a) Determine the maximum height of the disk.

b) State the domain and range for this function.

5. In the photograph, the fisherman is holding his fishing rod 0.5 m above the water. The fishing rod reaches its maximum height 1.5 m above and 1 m to the left of his hand.

a) Determine the quadratic function that describes the arc of the fishing rod. Assume that the y-axis passes through the fisherman's hand and the x-axis is at water level.

b) State the domain and range for the function that models the fishing rod.

6. Determine, to the nearest hundredth, the coordinates of the vertex of the following quadratic function:

$$q(x) = 0.4x^2 + 5x - 8$$

Lesson 6.3

7. a) Rewrite the following quadratic function in factored form:

$$f(x) = 2x^2 - 12x + 10$$

b) Identify the zeros of the function, and determine the equation of the axis of symmetry of the parabola it defines.

c) State the domain and range of the function.

d) Graph the function.

8. Determine the x-intercepts of the graph of this quadratic function:

$$f(x) = 2x^2 - 5x - 12$$

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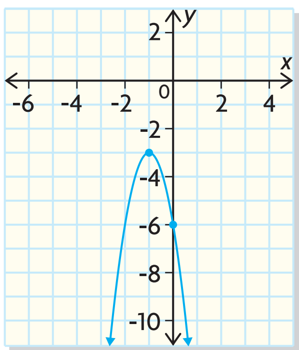
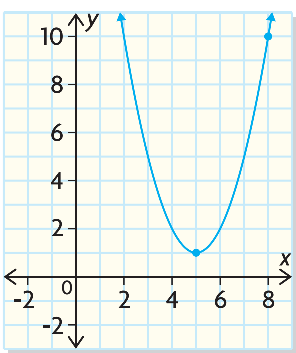
9. Determine the vertex of the parabola that is defined by each quadratic function. Explain your process.
   a) $f(x) = 3x^2 - 6x + 5$
   b) $g(x) = -1(x + 2)(x + 3)$

Lesson 6.4

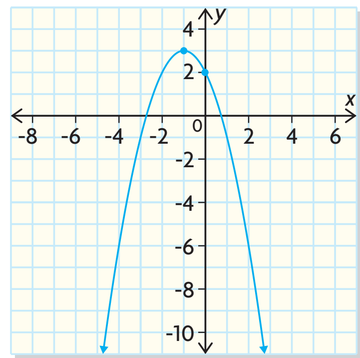
10. Determine the quadratic function, written in vertex form, that defines each of these parabolas.

a) 

b) 

11. a) State the direction of opening of the parabola that is defined by the following quadratic function:
$$y = 2(x - 3)^2 - 7$$
   b) Provide the equation of the axis of symmetry and the coordinates of the vertex of the parabola.
   c) State the domain and range of the function.
   d) Sketch the parabola.

12. Determine the quadratic function that defines this parabola:



13. Determine the quadratic function with zeros of -4 and -2, if the point $(-1, -9)$ is also on the graph of this function.

14. Determine the quadratic function that defines the parabola that has a vertex at $(3, -5)$ and passes through $(-1, -9)$.

15. The High Level Bridge in Edmonton is the source of the Great Divide Waterfall, which is open to the public on holiday weekends in the summer. The water falls a vertical distance of 45 m from the bridge and reaches the North Saskatchewan River 10 m horizontally from the base of the bridge. Determine a quadratic function that models the path of the water.

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Lesson 6.5

16. The Ponte Juscelino Kubitschek in Brasilia, Brazil, has three identical parabolic arches as shown above. Each arch is 61 m high and spans 219 m at a height of 10 m above the water.

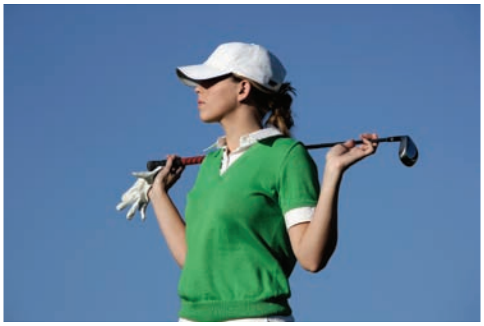
a) Determine a quadratic function that models one of the arches.

b) What other information would you need to determine the total span of the bridge?

17. A two-lane highway runs through a tunnel that is framed by a parabolic arch, which is 20 m wide. The roof of the tunnel, measured 4 m from its right base, is 4 m above the ground. Can a truck that is 4 m wide and 5 m tall pass through the tunnel?

18. Marcelle has ordered 80 m of stacking stones so that she can create a raised garden. She needs to place the stones on only three sides, since the garden will be built against her fence. Determine the dimensions that will enable Marcelle to maximize her planting area, and determine the maximum area of the garden.

19. On the 13th hole of a golf course, Saraya hits her tee shot to the right of the fairway. Saraya estimates that she now has 130 yd to reach the front of the green. However, she needs to clear some pine trees that are 40 yd from the green. The trees are about 10 yd high. Determine two different quadratic equations that model the flight of a golf ball over the trees and onto the green. Write one of your functions in factored form and the other in standard form.



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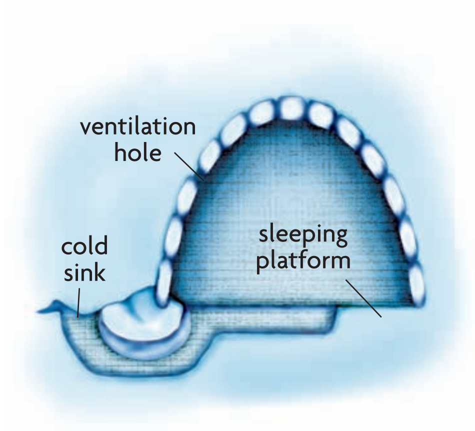
Parabolas in Inuit Culture

Traditional Inuit homes include arched domes, called igloos, that are made of snow blocks. The internal design of an igloo makes use of the principle that hot air rises and cold air sinks. In a typical igloo, hot air from the qulliq (seal oil lamp) and from human bodies rises and is trapped in the dome. Cold air sinks and is pooled at the entrance, which is lower than the living area. There are ventilation holes for the release of carbon dioxide.



? How can quadratic functions be used to model the cross-section of an igloo?

A. From the Internet or another source, obtain a picture of an igloo that appears to have a parabolic cross-section that you can use to estimate the size of an igloo.

B. Draw and label cross-sectional models of the interior of an igloo. Assume that each snow block is 1 ft thick. Also assume that the entrance to the igloo is 2 ft below the snow line.

C. Model the cross-sections of the arches on the exterior and interior of the igloo using quadratic functions, assuming that both arches are parabolic. State the domain and range of each function.

D. Based on your quadratic models, how close to the entrance of the igloo can a person who is 5 ft 6 in. tall stand, without ducking or bending?

Task *Checklist*

- ✔ Is your model neatly drawn and labelled?
- ✔ Have you stated and justified all the assumptions that support your quadratic models?
- ✔ Have you used appropriate mathematical language?

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6 Project Connection

Identifying Controversial Issues

While working on your research project you may uncover some issues on which people disagree. To decide on how to present an issue fairly, consider some questions you can ask yourself or others as you carry out your research.

1. What is the issue about?

Identify which type of controversy you have uncovered. Almost all controversy revolves around one or more of the following:

- Values—What should be? What is best?
- Information—What is the truth? What is a reasonable interpretation?
- Concepts—What does this mean? What are the implications?

2. What positions are being taken on the issue?

Determine what is being said and whether there is reasonable support for the claims being made. You can ask questions of yourself and of others as you research to test the acceptability of values claims:

- Would you like that done to you?
- Is the claim based on a value that is generally shared?

If the controversy involves information, ask questions about the information being used:

- Is there adequate information?
- Are the claims in the information accurate?

If the controversy surrounds concepts, look at the words being used:

- Are those taking various positions on the issue all using the same meanings of terms?

3. What is being assumed?

Faulty assumptions reduce legitimacy. You can ask:

- What are the assumptions behind an argument?
- Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
- Is the person presenting a position or opinion an insider or an outsider?

Insiders may have information and understanding not available to outsiders; however, they may also have special interests. Outsiders may lack the information or depth of understanding available to insiders; however, they may also be more objective.

4. What are the interests of those taking positions?

Try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their positions.

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Project Example — Identifying a controversial issue

Sarah chose the changes in population of the Western provinces and the territories over the last century as her topic. Below, she describes how she identified and dealt with a controversial issue.

Sarah's Explanation

I found that population growth can involve controversial issues. One such issue is discrimination in immigration policy. When researching reasons for population growth in Canada, I discovered that our nation had some controversial immigration policies.

From 1880 to 1885 about 17 000 Chinese labourers helped build the British Columbia section of the trans-Canada railway. They were paid only half the wage of union workers. When the railway was finished, these workers were no longer welcome. The federal government passed the *Chinese Immigration Act* in 1885, putting a head tax of $50 on Chinese immigrants to discourage them from staying in or entering Canada. In 1903, the head tax was raised to $500, which was about two years' pay at the time. In 1923, Canada passed the *Chinese Exclusion Act*, which stopped Chinese immigration to Canada for nearly a quarter of a century. In 2006, Prime Minister Stephen Harper made a speech in the House of Commons apologizing for these policies.

I decided to do some more research on Canada's immigration policies, looking at historical viewpoints and the impact of the policies on population growth in the West and North. I will include a discussion about the effects of these policies in my presentation and report.

Your Turn

A. Identify the most controversial issue, if any, you have uncovered during your research.

B. Determine the different positions people have on this issue and the supporting arguments they present. If possible, include any supporting data for these different positions.

C. If applicable, include a discussion of this issue in your presentation and report.

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